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Wide-angle Microwave Lens for Line Source Applications

W. ROTMAN
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ABSTRACT

The design equations for a constrained wide-angle, two-dimensional microwave lens have been derived for the special case in which the front lens face is straight and in which lens elements can connect arbitrary points on the two lens contours. A phase analysis indicates that this lens design has very small coma aberrations and is capable of generating beams a fraction of a degree in width.

Criteria are developed for selection of optimum lens parameters for specific applications.

An experimental model in which the lens elements consist of coaxial cables was constructed to demonstrate techniques of fabrication. Radiation patterns indicated the expected characteristics. The application of these principles to the design of symmetrical three-dimensional lenses is briefly indicated.

Tables of lens contour parameters and path length aberrations are presented for the specific case of a scan angle α of 30°.

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WIDE-ANGLE MICROWAVE LENS FOR LINE SOURCE APPLICATIONS

1. INTRODUCTION

The wide-angle scanning characteristics of two-dimensional microwave lenses have been extensively investigated[1] and applied to the design of radar antennas. Ruze[2] has shown, for example, that constrained lenses of the no-second-order type are capable of generating 1° beamwidths and of scanning these beams over angles as great as one hundred beamwidths. The no-second-order designation refers to a lens with two perfect off-axis symmetrical focal points and an on-axis focal point for which the second-order phase deviation is zero. For no-second-order lenses both front and back lens faces are curved.

Ruze also discusses the design of a straight-front-face lens required for line source applications as the primary illuminator for a parabolic cylindrical reflector or as the feed for a rectangular planar array. This straight-face lens has excellent scanning characteristics since both second and third order coma may be almost eliminated by proper defocussing. It has two perfect, symmetrical, off-axis focal points and a highly corrected on-axis focal point. For very narrow beam antennas, however, its higher order coma aberrations may still be objectionable.

A further improvement in coma aberrations may be achieved by applying general lens design principles, developed by Gent et al.[3,4], to the special case of the straight-front-face lens. This results in a lens design with three perfect focal points, two symmetrically located off-axis and one on-axis. It is the purpose of this paper to obtain the design equations for the improved straight-front-face lens from Gent's generalized equations, to evaluate its phase aberrations and scanning capabilities, and to demonstrate fabrication techniques applicable to this type of design.

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2. THEORY

2.1 Derivation of Design Equations

A two-dimensional schematic representation of the straight-front-face lens is shown in Fig. 1. The basic external difference between this lens and Ruze's design is that the lens elements are lengths of coaxial transmission line, rather than waveguide. This permits the connection of arbitrary points on the front and rear surface of the lens so that corresponding front and rear surface distances, N and Y, for a single lens element, are not necessarily equal (as they are in Ruze's design). This additional degree of freedom permits specifying four independent conditions to determine the lens uniquely, rather than the three conditions that were available to Ruze. In the present lens design, these conditions were selected as the straight-front face, the two symmetrical off-axis focal points, and an on-axis focus. The ends of the coaxial cables that form the transmission line elements of the microwave lens are connected directly to a straight line of radiators to form a line source.

The formulation of the lens design equations and notation follows that of Gent.[3] In Fig. 2, the lens surfaces are shown two-dimensionally by the cross sections Σ_1 and Σ_2. The first contour, Σ_1, determines the position of the probe transitions between the parallel plates and the coaxial cables. The second contour, Σ_2, is straight and defined by the location of the radiating elements that comprise the line source. Corresponding elements on contours Σ_1 and Σ_2 are connected by a transmisson line TL.

The contour Σ_1 is defined by the two coordinates (X, Y) that are measured relative to a point 0_1 on the central axis of the lens. Points on the straight contour Σ_2 are similarly determined by the single coordinate N, measured relative to the point 0_2 on the axis. The points 0_1 and 0_2 lie on contours Σ_1 and Σ_2 respectively and are connected by a transmission line TL_0 of electrical length W_0. The point P, defined by the coordinates X and Y, is a typical probe element in Σ_1 and is connected to point Q, which lies on Σ_2 and is defined by the coordinate N, by the transmission line TL of electrical length W. The three quantities X, Y, and W can be chosen at will; thus this straight-front-face lens has

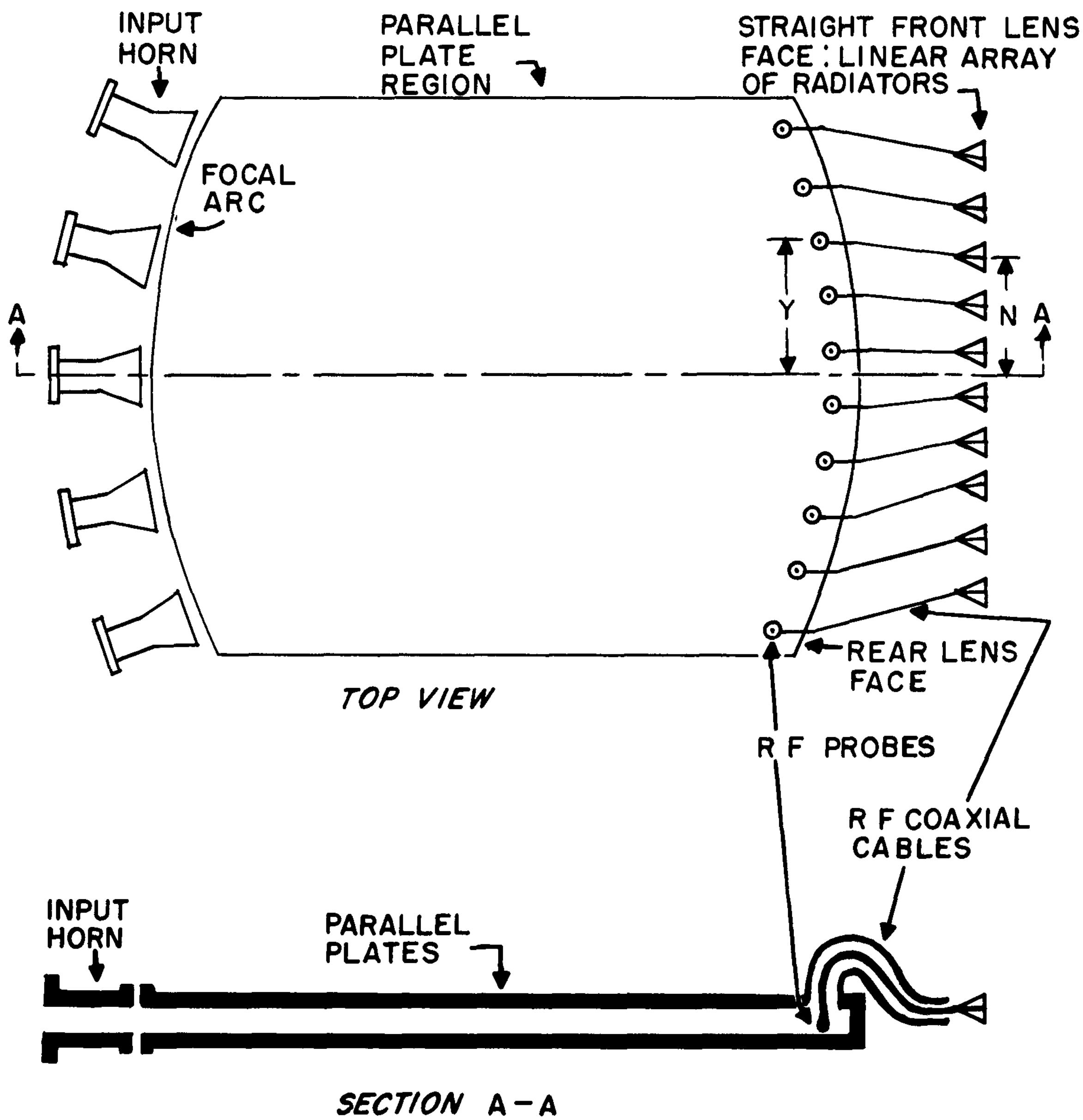

FIG. 1. Parallel-plate microwave lens.

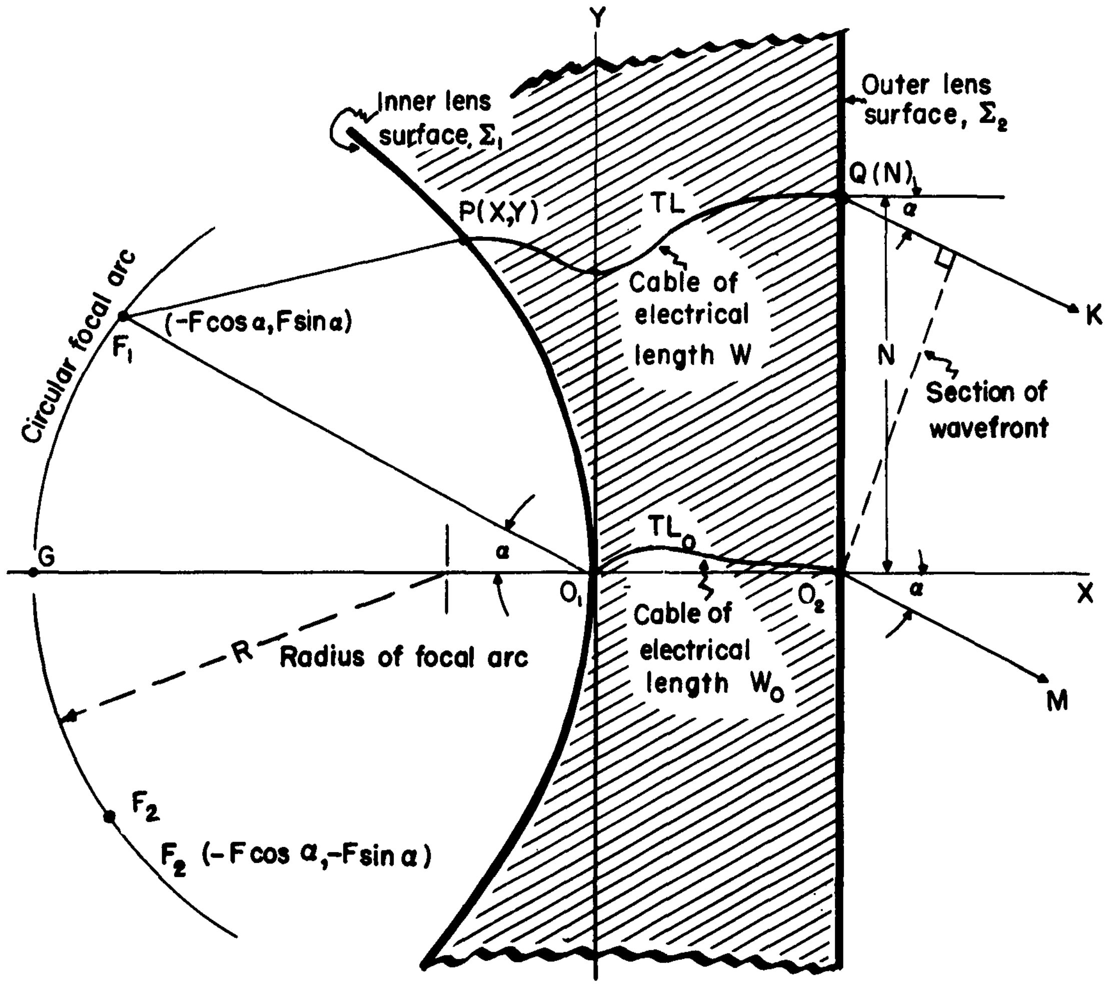

FIG. 2. Ray trace diagram for two-dimensional electromagnetic lens.

three degrees of freedom. Other types of lenses, (including Ruze's design in which $Y = N$) have at least one less degree of freedom.

Actual values for the three degrees of freedom will now be selected to obtain wide-angle scanning characteristics. These design parameters include (Fig. 2) two symmetrical off-axis focal points, F_1 and F_2, and one on-axis focal point, G, having coordinates $(-F\cos\alpha, F\sin\alpha)$, $(-F\cos\alpha, -F\sin\alpha)$, and (G, O) respectively relative to the point 0. A ray through the lens at the origin is represented by $\overline{F_1 0_1 0_2 M}$ and $\overrightarrow{F_1PQK}$ represents any other typical ray.

The lens is now designed so that the three focal points F_1, F_2, and G give perfectly collimated beams of radiation at angles to the axis of $-\alpha$, $+\alpha$, and $0°$ respectively.

In our special case Gent's[3] equations for the optical path-length conditions for path-length equality between a general ray and the ray through the origin are:

$$(F_1P) + W + N\sin\alpha = F + W_0 \quad , \tag{1}$$

$$(F_2P) + W - N\sin\alpha = F + W_0 \quad , \tag{2}$$

and

$$(G_1P) + W = G + W_0 \; , \tag{3}$$

where

$$(F_1P)^2 = F^2 + X^2 + Y^2 + 2FX\cos\alpha - 2FY\sin\alpha \quad , \tag{1a}$$

$$(F_2P)^2 = F^2 + X^2 + Y^2 + 2FX\cos\alpha + 2FY\sin\alpha \quad , \tag{2a}$$

and

$$(G_1P)^2 = (G + X)^2 + Y^2 \; . \tag{3a}$$

F_1P, F_2P, and G_1P represent path lengths from focal points F_1, F_2, and G respectively to the rear surface of the lens.

We now normalize relative to the focal length F by defining a new set of parameters

$$\eta = N/F, \quad x = X/F, \quad y = Y/F,$$

$$w = \frac{W - W_0}{F}, \quad g = G/F.$$

Also

$$a_o = \cos \alpha,\ b_o = \sin \alpha.$$

Equations (1a) to (3a) may then be written

$$\frac{(F_1X)^2}{F^2} = 1 + x^2 + y^2 + 2a_o x - 2b_o y, \tag{1b}$$

$$\frac{(F_2P)^2}{F^2} = 1 + x^2 + y^2 + 2a_o x + 2b_o y, \tag{2b}$$

and

$$\frac{(GP)^2}{F^2} = (g + x)^2 + y^2. \tag{3b}$$

Combining the normalized Eqs. (1) and (1b)

$$\begin{aligned} \frac{(F_1P)^2}{F^2} &= (1 - w - b_o \eta)^2 \\ &= 1 + w^2 + b_o^2 \eta^2 - 2b_o \eta + 2b_o w \eta \qquad (1c) \\ &= 1 + x^2 + y^2 + 2a_o x - 2b_o y. \end{aligned}$$

Since the off-axis focal points are symmetrically located, the lens surfaces must also be symmetrical about the center axis. This means that, if η is replaced by $-\eta$ and y by -y, Eq. (1c) remains unchanged. Equation (1c) can therefore be separated into two independent equations; one contains only odd powers of y and η while the other contains the remaining terms. Thus,

$$-2b_o \eta + 2b_o w \eta = -2b_o y$$

or (4)

$$y = \eta\ (1-w).$$

Also

$$x^2 + y^2 + 2a_0 x = w^2 + b_0^2 \eta^2 - 2w. \tag{5}$$

Equations (3) and (3b), relating to the on-axis focus, may likewise be written

$$\frac{(GP)^2}{F^2} = (g - w)^2 = (g + x)^2 + y^2 \tag{3c}$$

or

$$x^2 + y^2 + 2gx = w^2 - 2gw. \tag{6}$$

Equations (5) and (6) can be combined to give the following relation between w and η

$$aw^2 + bw + c = 0, \tag{7}$$

where

$$a = \left[1 - \eta^2 - \left(\frac{g - 1}{g - a_0}\right)^2\right],$$

$$b = \left[2g\left(\frac{g - 1}{g - a_0}\right) - \left(\frac{g - 1}{g - a_0}\right)^2 b_0^2\, \eta^2 + 2\eta^2 - 2g\right],$$

$$c = \left[\frac{g\, b_0^2 \eta^2}{g - a_0} - \frac{b_0^4\, \eta^4}{4(g - a_0)} - \eta^2\right].$$

Equation (7) is a quadratic equation in w whose solution is

$$w = \frac{-b + \sqrt{b^2 - 4ac}}{2a}.$$

This completes the solution for the lens design. For fixed values of α and g, w can be computed as a function of η from Eq. (7). These values of w and η may then be substituted into Eqs. (4) and (6) to determine x and y and complete the specification of the lens dimensions.

2.2 Selection of Optimum Parameters

The design procedure, as outlined, gives a lens which has three perfect focal points, corresponding to the angles $\pm\alpha$ and $0°$. For wide angle scanning, the lens must focus well, not only at these three points, but also at intermediate angles along some focal arc. The basic lens equations do not indicate how to select the remaining variables, such as the factor g (ratio of on-axis focal length G to off-axis focal length F) to minimize the overall phase aberrations.

A clue to the optimum value of the parameter g may be obtained from Ruze's phase error analysis of an electromagnetic wave lens under the restrictions that $y = \eta$. His basic design assumes a lens in which the focal arc is a portion of a circle of radius F, centered at the origin of the surface Σ_1. For the special case of a straight-front-face lens, he shows that minimum coma and overall phase error is obtained by defocussing the feed from the assumed focal arc by an amount equal to $1/2\,(\alpha^2 - \theta^2)\,F$ where θ is the intermediate angle at which correction is desired. With this defocussing the residual aberrations are quite small and the lens can scan a narrow beam over wide angles.

Under the above conditions, Gent's ($y \neq \eta$) and Ruze's ($y = \eta$) designs for a straight-front-face lens should have closely related parameters since in the former case the on-axis aberrations are at zero while in the latter case these aberrations are at a minimum. It would therefore seem reasonable to select a value of g in our derivation that corresponds to this same amount of defocussing ($\theta = 0°$). This establishes the optimum value of g as:

$$g = \frac{G}{F} = 1 + \frac{\alpha^2}{2}. \tag{8}$$

The focal arc is now selected (Fig. 3) as a segment of a circle of radius R, whose center lies on the axis of symmetry of the lens and which passes through the two symmetrical off-axis and the one on-axis focal points. The phase error from any point on this focal arc (expressed as

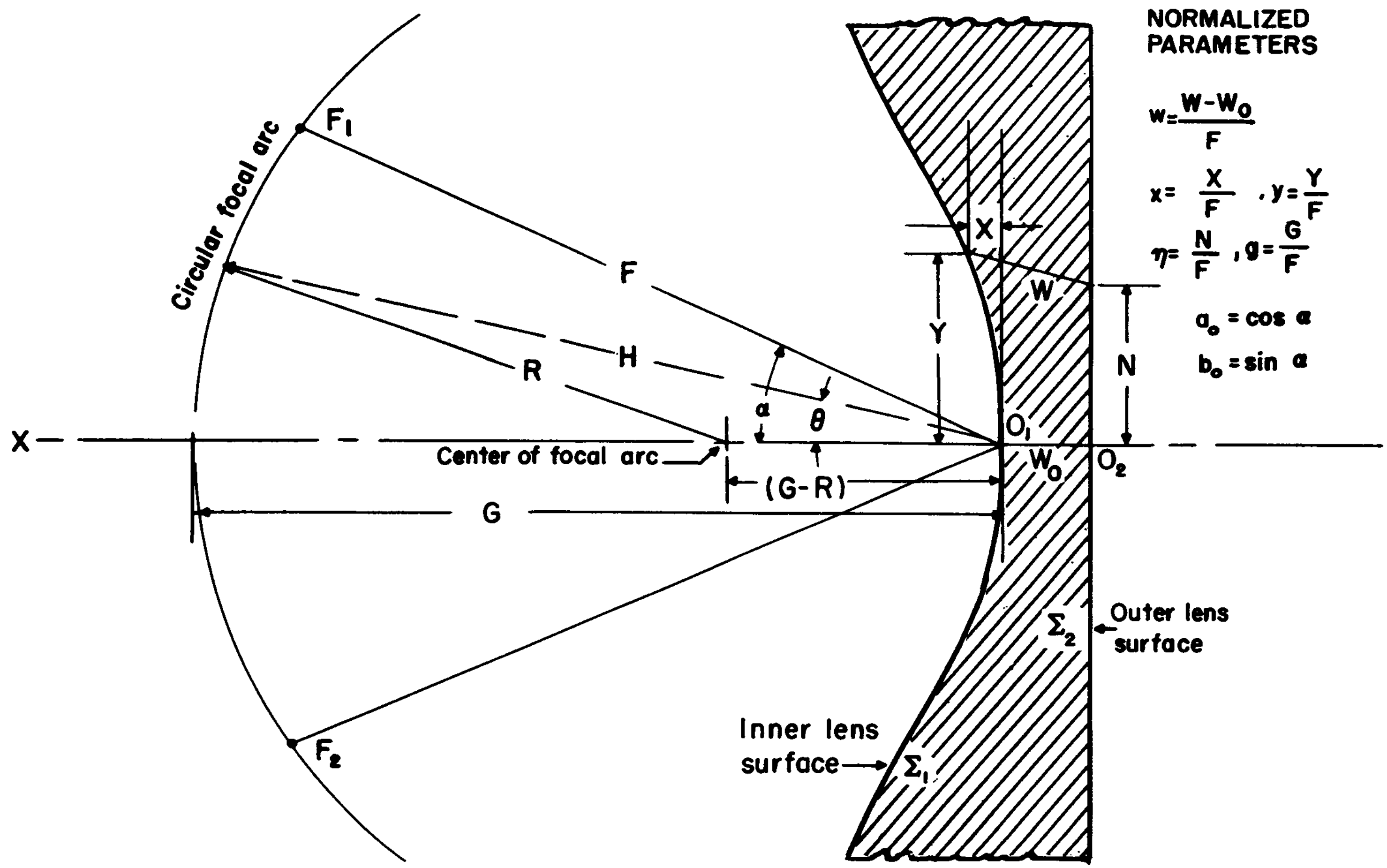

FIG. 3. Microwave lens parameters.

the difference in path length between the central ray and any other ray) may be shown to be:

$$\Delta \ell = \frac{\Delta L}{F} = (h^2 + x^2 + y^2 + 2hx\cos\theta - 2hy\sin\theta)^{1/2} - h + w + \eta\sin\theta \qquad (9)$$

where

ΔL = path length error,

$h = H/F$ = normalized distance from point on focal arc to origin 0_1 of surface Σ_1. H is determined from the triangle with sides R, H, and G-R and with included angle θ (Fig. 3).

θ = angle between central axis and point on focal arc.

R = radius of focal arc (determined by the three points G, F, and F_2 on the arc).

2.3 Lens Contour and Phase Error Calculations

The preceding section indicated that an optimum set of lens parameters exists, in the sense that they provide minimum phase aberrations over a prescribed range of angles. This selection of parameters assumes that the correction for second and third order coma terms also results in the minimization of the higher order aberrations. Since this is not obvious, an investigation was conducted to determine the phase errors in lenses of this type for parameters that may differ from the optimum value.

The total scan angle, 2α, for the lens was fixed at 60° (α = 30°), as representative of a wide-angle lens system. From Eq. (8), the optimum on-axis focal point is given by g = 1.137. Accordingly, calculations were made on a digital computer of the normalized lens contour parameters, y, x, and w and also of the normalized phase error, $\Delta \ell$, from Eqs. (4), (6), (7), and (9) for the following range of values:

$g = 0.90 \langle 0.05 \rangle 1.20$, and 1.137.

$\theta = \pm 5°, \pm 15°, \pm 25°, \pm 35°$.

$\eta = 0 \langle 0.25 \rangle 0.80$.

For $\theta = 0°$ and $\pm 30°$, $\Delta \ell$, is zero.

Selected lens contour curves are shown in Figs. 4<u>a</u> to <u>d</u> for g = 1.00, 1.10, 1.137, and 1.20. Their tabular values are given in

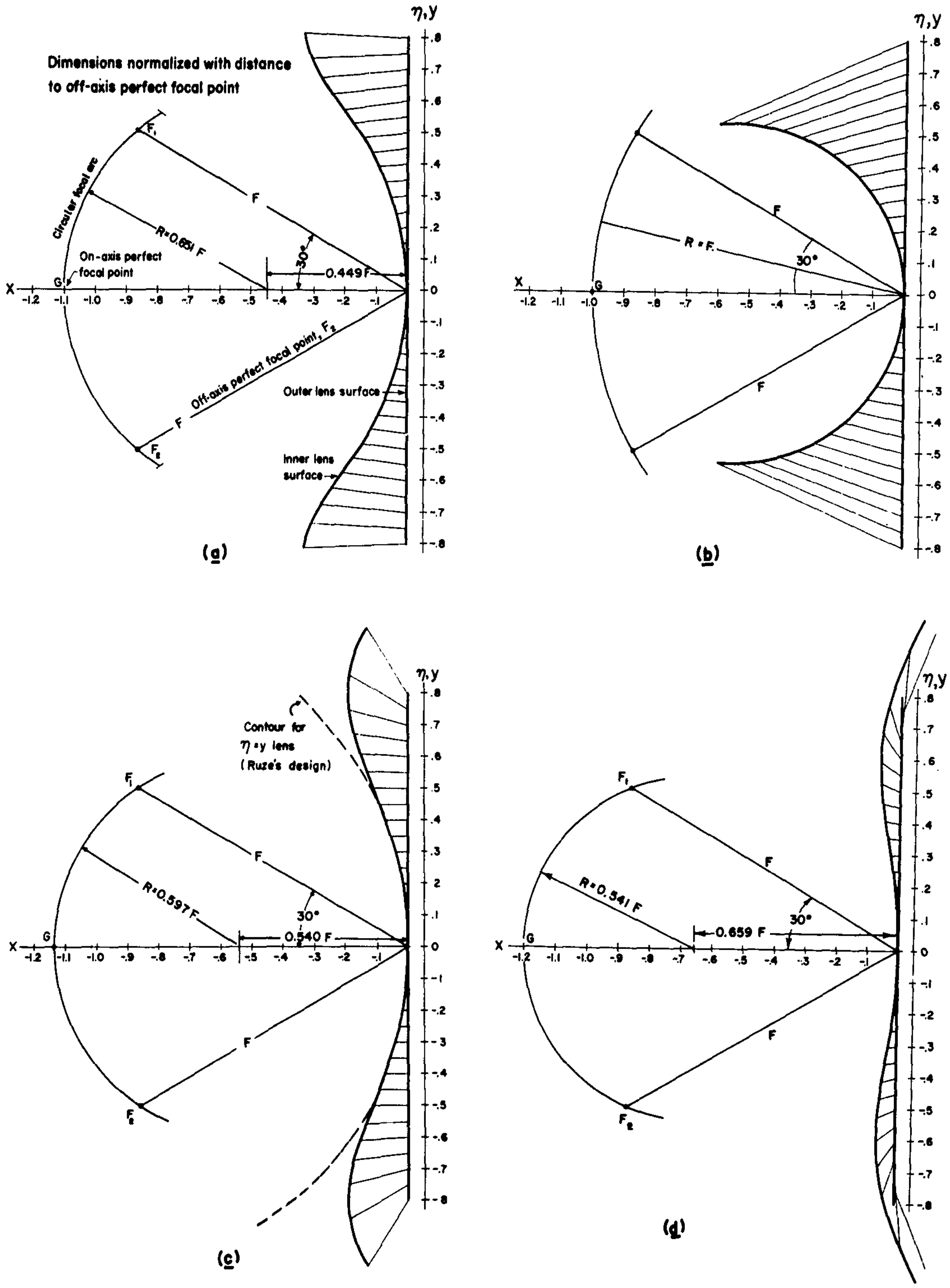

FIG. 4. Microwave lens contours, (a) G = 1.10F; (b) G = F; (c) G = 1.137F; (d) G = 1.20F.

Appendix A for $g = 0.90 \langle 0.05 \rangle 1.20$ and 1.137. The light lines between the rear and front lens contours indicate corresponding values of y and η which are the junction points for the coaxial lens elements.

A comparison is made in Fig. 4c between the rear lens contour for the Gent ($y \neq \eta$) and the Ruze ($y = \eta$) designs for the optimized value of $g = 1.137$. In the latter case the straight-front-face lens equations are

$$x^2 + a_0^2 y^2 + 2a_0 x = 0, \qquad w = 0, \text{ and } \eta = y. \tag{10}$$

The rear lens contour is therefore elliptical and does not depend on the value of g. It should be noted that the lens contours for both $y \neq \eta$ and $y = \eta$ conditions practically coincide for values of η less than 0.65. This agrees with our original assumption.

The lens contour for which $g = 1.00$ (Fig. 4b) is also of interest in that it is the only case for which the focal arc is centered at the vertex of the rear lens face and for which the central ray paths from all points on the focal arc are equal in length. Such considerations may be of importance in monopulse applications. This lens has been extensively investigated by Hatcher[5] who showed that its inner contour could be approximated by a segment of a circle. Its phase aberrations, however, are considerably poorer than those for the optimum design.

Phase aberrations, expressed in terms of the normalized difference in optical path length relative to the central ray $\Delta \ell$, are tabulated in Appendix B for $g = 0.90 \langle 0.05 \rangle 1.20$ and 1.137, and shown graphically in Fig. 5a to c for $g = 1.00$, 1.10, and 1.137. It can be seen that, for values of η less than ± 0.53, $g = 1.137$ is indeed optimum in the sense that the path length error, $\Delta \ell$, remains below ± 0.0001 for all angles of scan up to $\pm 35°$. If the permissible path length error, $\Delta \ell$, can be as great as ± 0.0005, however, a lens with $g = 1.10$ may be more suitable since it would permit apertures up to $\eta = 0.8$. Note that the path length errors for the $g = 1.00$ lens have essentially even sym-

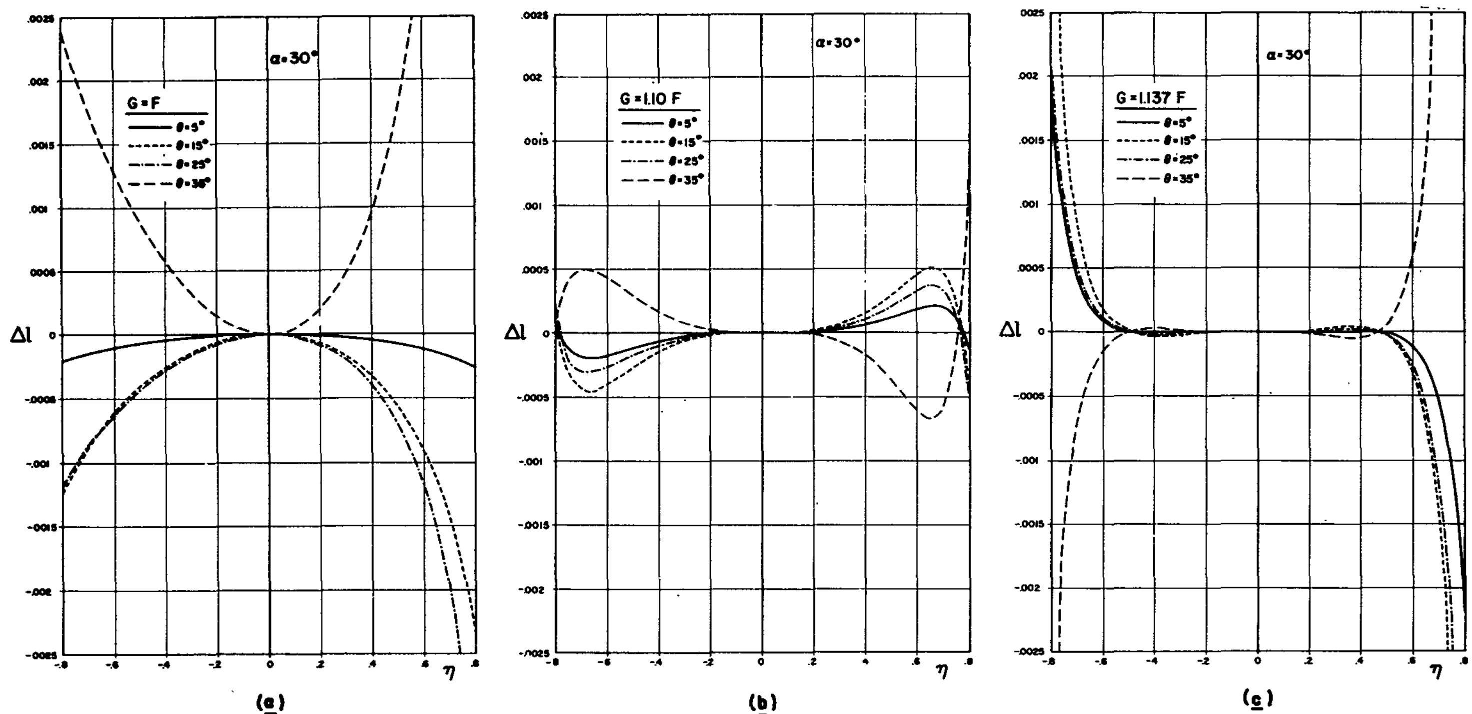

FIG. 5. Path length errors in microwave lens, (a) G = F; (b) G = 1.10F; (c) G = 1.137F.

metry with respect to η while those for g = 1.10 and g = 1.137 have odd symmetry. (The path length error curves show only positive values for θ while η takes on positive and negative values. Alternatively, both positive and negative values of θ could be used with only positive values of η.)

The minimum beamwidth obtainable from a microwave lens is determined by the size of the aperture, the operating wavelength, and the maximum permissible phase or path-length aberrations. The beamwidth for a rectangular aperture antenna with a cosine illumination taper and 23-db sidelobes (typical of the usual antenna practice) is given by

$$HPBW = 69° \times \frac{\lambda}{D} \tag{11}$$

where

HPBW is the half-power beamwidth,

D ($= 2N_{max} \cos \theta$) is the projected aperture,

λ is the wavelength, and

N_{max} is one-half of the physical aperture.

For minimum beamwidth, the maximum path-length error deviation ($\sim 2\Delta L_{max}$) cannot exceed about ¼λ without adversely affecting the sidelobe level.

Thus

$$(\Delta L)_{max} = \frac{\lambda}{8} \tag{12}$$

Also

$$D = 2N_{max} \cos \theta \tag{13}$$

Combining Eqs. (11), (12), and (13)

$$\frac{D}{\lambda} = \frac{\eta_{max} \operatorname{Cos} \theta}{4(\Delta l)_{max}} \tag{14}$$

and the minimum possible beamwidth for a given lens is

$$(HPBW)_{min} = \frac{(\Delta l)_{max}}{\eta_{max} \operatorname{Cos} \theta} \times 276°. \tag{15}$$

As a typical example, the following parameters are selected to give the minimum beamwidth:

$$g = 1.137,$$

$$\eta_{max} = 0.55,$$

$$G/D = \frac{g}{2\eta_{max}} = 1.035,$$

$$-30° \leqslant \theta \leqslant +30°,$$

and

$$(\Delta \ell)_{max} = 0.00013 \text{ (from Fig. 5c and Appendix B.}$$

Then:

$$(HPBW)_{min} = 0.065° \text{ (For } \theta = 0°)$$
$$= 0.075° \text{ (For } \theta = 30°).$$

Thus, we can scan a beam of less than 0.1° over 60° for a total scan angle of greater than 600 beamwidths.

As a second example, we will try to minimize the ratio G/D (focal length to aperture ratio) at the expense of a somewhat greater minimum beamwidth.

The following parameters are therefore chosen:

$$g = 1.100,$$
$$(\Delta \ell)_{max} = 0.0005,$$
$$-30° \leqslant 0 \leqslant +30°,$$
$$\eta_{max} = 0.80$$
$$G/D = 0.687 \text{(For } \theta = 0°),$$

and

$$(HPBW)_{min} = 0.18° (\theta = 0°)$$
$$= 0.21° \; (\theta = 30°).$$

For both these examples, the scan angle θ can be extended to ±35° with very little deterioration of radiation pattern. Note, also, that defocussing from the assumed circular focal arc to some other noncircular contour that also passes through F_1, F_2, and G does not accomplish much in reducing coma aberrations (except for the case

of g = 1.00) since defocussing primarily affects the even-order aberrations whereas the residual phase aberrations are primarily of odd order.

The g = 1.137 lens contour is thus optimum in the sense that it minimizes the obtainable beamwidth for a reasonable F/D ratio. The value of g = 1.10 seems more suitable, however, if the F/D ratio must be minimized and if the permissible beamwidth is greater than 1/4°. Either of these two designs should be equally applicable, however, to the majority of lens design problems.

3. EXPERIMENTAL STUDIES

3.1 Wide-Angle Lens Design

The analysis of the previous sections has shown that the design of wide-angle microwave lenses, with beamwidths on the order of fractions of a degree, is theoretically feasible. A two-dimensional model of such a microwave lens was constructed to determine the problems inherent in this design. Design specifications include the following parameters:

$f_o = 3.0$ Gc (design frequency),

$g = 1.137$,

$\eta_{max} = 0.60$,

$D/\lambda = 18$,

$\theta_{max} = 30°$,

HPBW = 3° (Nominal Half-Power Beamwidth),

Primary Illuminator—Open-Ended Waveguide Horn,

Lens Elements—RG-9/U Coaxial Cables.

Refer to Fig. 4c for the lens contour and to Fig. 5c for the normalized path length error $\Delta \ell$.

The beamwidth for this lens was chosen to keep its physical dimensions within reasonable limits. On the other hand, the theoretical phase errors inherent in this model are so small that they cannot be detected by their effect on the radiation pattern or other electrical characteristics of the lens. For example, either the Ruze ($y = \eta$) or Gent design for g = 1.00, 1.10 or 1.137 (Figs. 4 and 5) results in lenses with equivalent electrical performance when the beamwidth is approximately 3°.

Our objective is not to obtain an experimental comparison between the competitive Ruze and Gent lens designs, but rather to demonstrate techniques unique to the construction of microwave lenses with variably spaced coaxial lens elements.

The microwave lens model (Fig. 6) uses the parallel plate and coaxial TEM modes to obtain maximum bandwidth. Microwave radiation from the primary horn illuminators, located along the focal arc, propagates between the parallel plates to the reflector-backed probes that form the rear lens contour. These probes extract the energy from the parallel-plate region and feed it into the coaxial cable lens elements which, in turn, excite the probes on the straight-front-lens contour. These front probes form a linear array between a second set of parallel plates that radiate into space through a short TEM horn transition. The lens in Fig. 6 is shown feeding a cylindrical parabolic reflector that collimates the beam in the elevation plane.

The primary horn illuminator is designed for a prescribed amplitude distribution along the front lens face by selection of its aperture dimensions and orientation. Its required radiation pattern between the parallel plates is first determined graphically by ray-tracing, equating the power radiated per unit length along the front lens face to the angular sector subtended by this power flow at the horn position on the focal arc. The aperture and orientation of the horn illuminator are then selected[7] to give the closest approximation to the desired primary pattern. The realizable lens illumination differs somewhat from the theoretical value because the required primary pattern is not symmetrical (except for the on-axis position $\theta = 0°$) and does not conform exactly to the physically realizable patterns obtained from a uniformly-phased horn illuminator. The farfield radiation pattern is, however, not very sensitive to small changes in illumination taper. Since the input horn's directivity is a function of its position along the focal arc, its dimensions and orientation depend upon its focal position. Also, the peak of radiation is not, in general, in the direction of the vertex of the lens. For example, the required horn parameters in the present model vary from an aperture of 1.2λ (primary HPBW = 41°) for the on-axis focal point

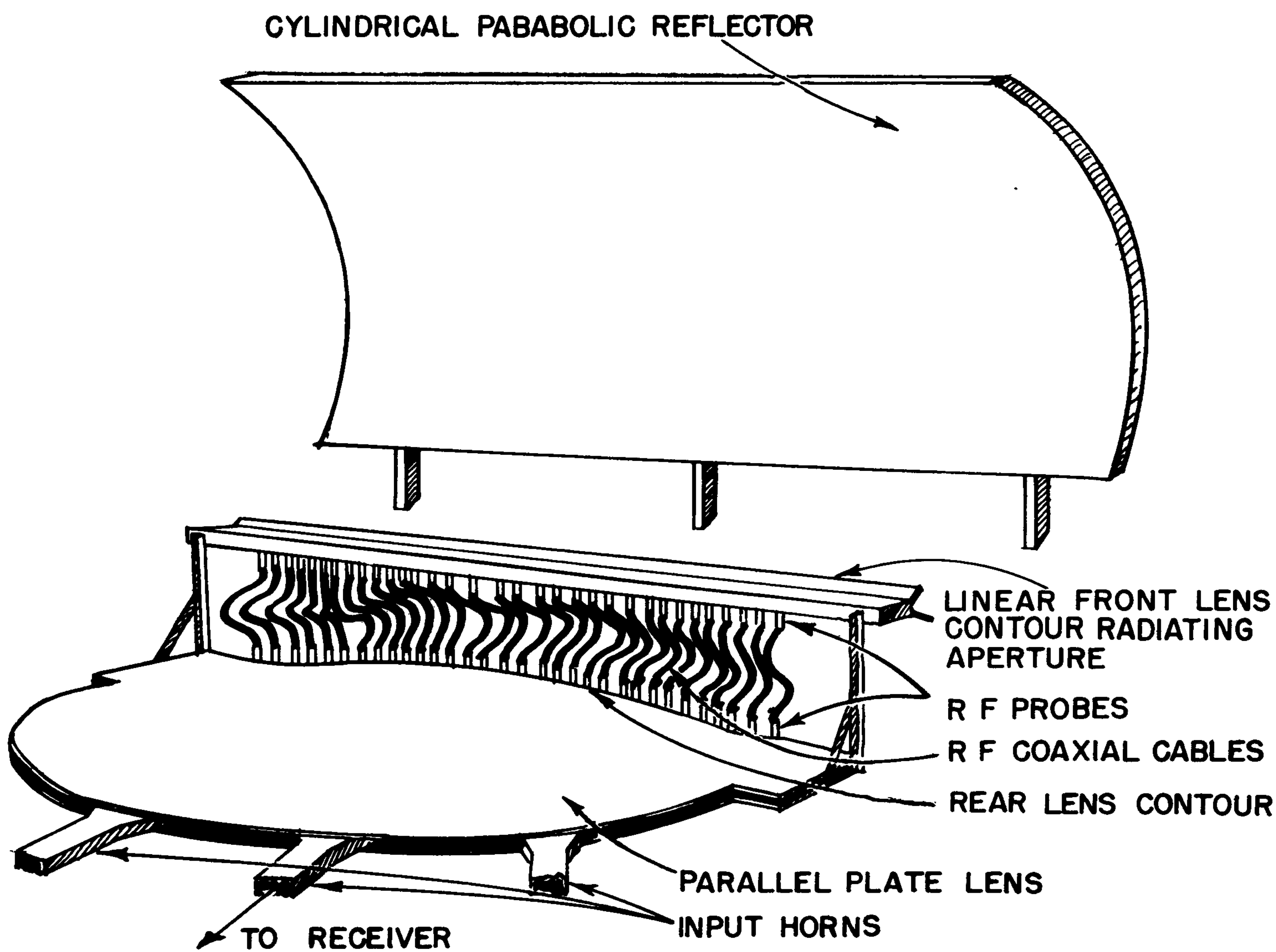

FIG. 6. Parallel-plate lens as line source feed for reflector.

to 1.45λ (primary HPBW = 34°) for the $\theta = 30°$ focal point. Since several stationary horn illuminators are used in the experimental model, this change in horn dimensions causes no difficulty. If the beam scanning were accomplished by moving a single input horn along the focal arc, a compromise aperture dimension (for example, 1.3λ) would have to be selected.

The dimensions and spacing of the probes must be chosen to ensure adequate coupling of the lens elements to the parallel plate structures over a wide range of incident angles. The performance of each probe is affected by mutual coupling to its neighbors. The problem of analyzing the behavior of a set of probes located along a curved contour and excited by a curved wavefront is quite difficult. An approximate evaluation can be obtained, however, by considering the performance of an infinitely long linear array of uniformly spaced probes phased to radiate a plane wave front at an angle, γ, normal to the array. The relation between this angle of radiation and the electrical phasing of the probes, φ, is given by

$$\varphi = \frac{2\pi d}{\lambda_o} \sin\gamma \pm 2\pi n; \; n = 0, 1, 2, 3, \text{ and so forth,} \tag{16}$$

where d is the spacing between probes.

Equation (16) has multivalued solutions for γ when d/λ_o is large, corresponding to the generation of secondary maxima in the diffraction pattern. This condition must be avoided by restricting the probe spacing to values of d/λ_o given by

$$d/\lambda_o \leqslant \frac{1}{|\sin\gamma_o| + 1}, \tag{17}$$

where γ_o is the solution of Eq. (16) with $n=0$. This relation assures that Eq. (16) is single-valued in γ and has a real solution for $n = 0$ only. The probe spacing is therefore restricted to spacings of less than λ for broadside radiation ($\gamma_o = 0°$) and to 1/2 λ for endfire radiation ($\gamma_o = 90°$).

The restrictions of Eq. (17) are somewhat more stringent than required for the experimental model since this equation applies to a set of omnidirectional probes. The limited range of the angle of radiation, γ, restricted by the dipole-like element pattern of the reflector-backed probes to within about $\pm$ 60°, permits a probe spacing of about $0.65\lambda_0$ rather than the value of $0.535\lambda_0$ indicated for a maximum value for γ_0 of 60°. The probe spacing along the straight-front-lens face, held at a constant value of 0.50λ, places the coaxial elements at equal increments of the parameter η. This results in unequal spacing of the coaxial elements and probes along the rear lens face (since $y = \eta$). This spacing is always less than the permissible probe spacing of 0.65λ.

The probe dimension is determined by constructing a section of an array of probes, backed by a straight reflecting surface, between parallel plates. Tests were made of the input impedance of a single probe when all the other probes were terminated in matched loads and also of the power reflected from this array for angles of incidence of a plane wave of up to 60°. It was found that the best probe impedance match and the minimum reflected power occur at the same values of probe parameters. The probes penetrate 0.15λ and are located about 0.21λ from the reflector; the probe spacing and the parallel plate spacing are 0.5λ and 0.375λ respectively. It should be noted that the spacing between probes determines the probe penetration and distance from the reflecting surface.

The theoretical lens contours coincide with the reflecting surface behind the probes, rather than with the position of the probes themselves, since the phase center of radiation for a probe imaged in a ground plane is located at the surface of the reflector. The reflecting surface for the rear lens contour was therefore determined from the theoretical values of x and y. The probe position is located 0.21λ from this contour (measured along its normal). Front and rear views of the completed microwave lens are shown in Figs. 7 and 8. Several horns are used simultaneously to give a number of independent beams. The flexibility of the coaxial cable lens elements permits displacement

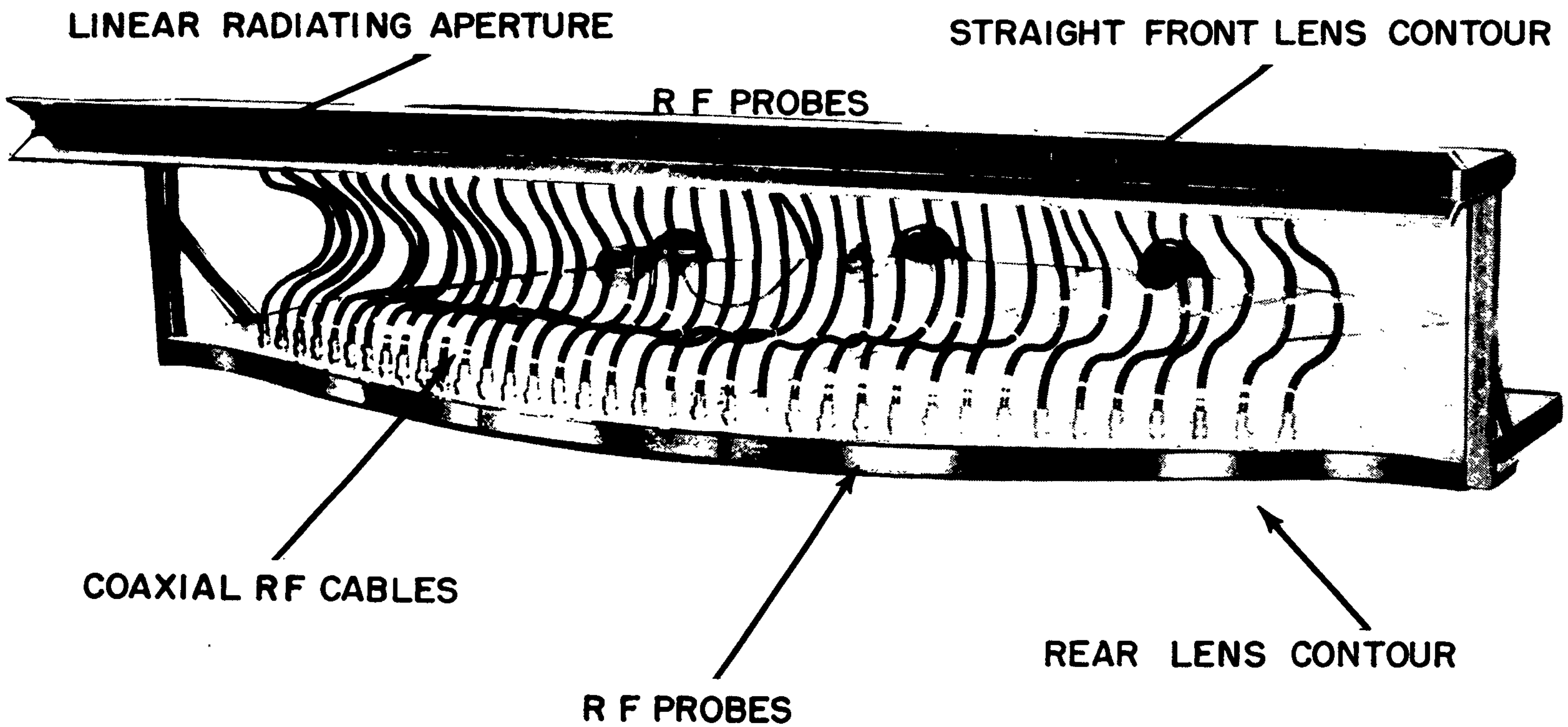

FIG. 7. Experimental microwave lens antenna. (Front View)

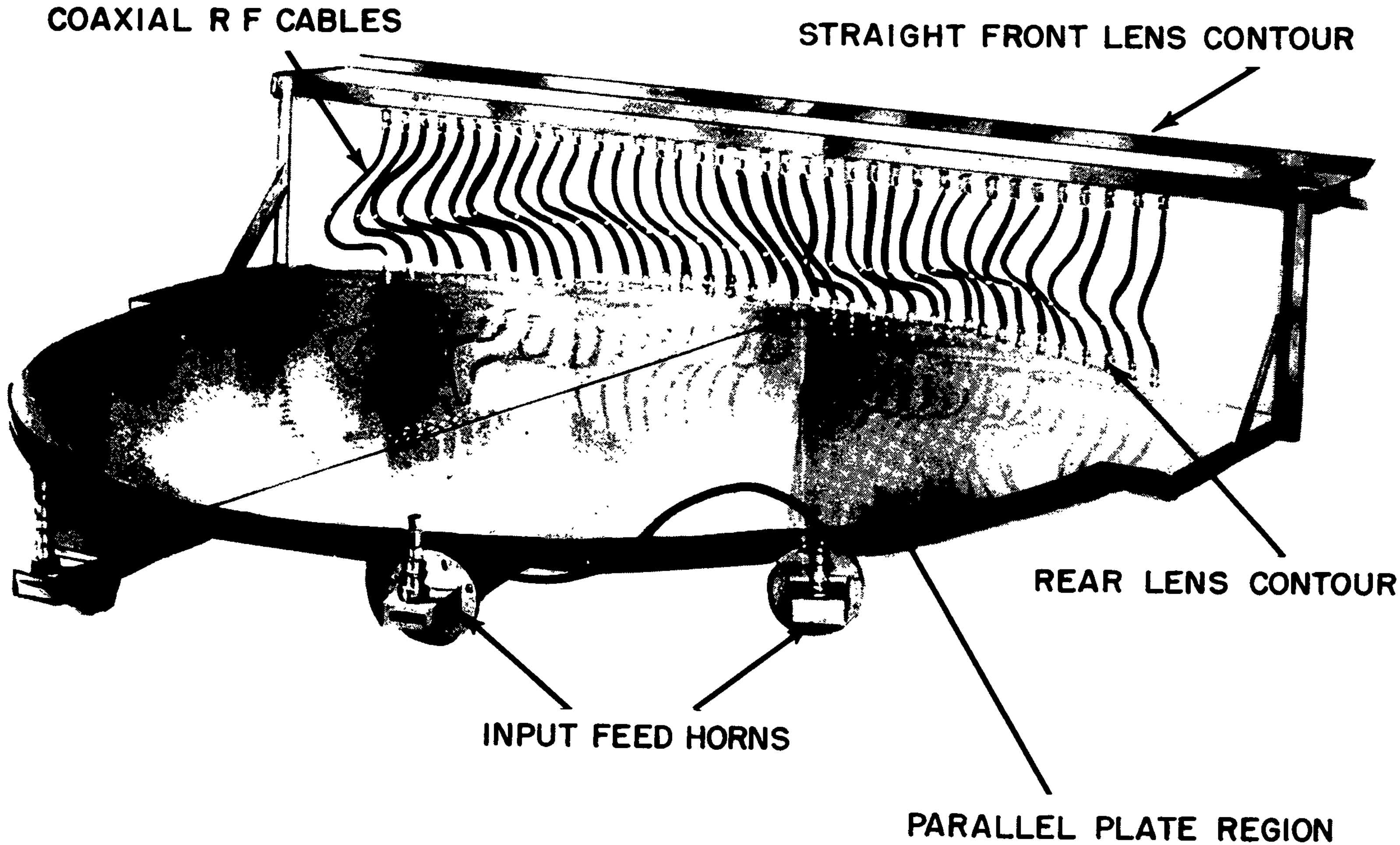

FIG. 8. Experimental microwave lens antenna. (Rear View)

of the line source, which forms the straight front face of the lens, from the rest of the lens structure.

3.2 Radiation Characteristics of Lens Model

Many radiation patterns of this microwave lens were measured for different angular positions and sizes of the horn illuminators and for focussing adjustments over a range of frequencies. Some representative patterns are shown in Figs. 9 a to c for a frequency of 3 Gc and for $\theta = 0°$, $15°$, and $30°$. The horn was located on the focal arc and had an aperture of 1.45λ. In all cases the position of best focus was found to be within one inch of the focal arc and was not particularly critical. The radiation patterns were also measured over the frequency range of 2.8 to 3.2 Gc with no significant deterioration of performance. The measured sidelobe levels, as high as −18 db, are somewhat inferior to the −22 db design value. This deviation is probably caused by a nonoptimum selection of horn aperture size (which would also account for the measured beamwidth being somewhat smaller than the theoretical value), as well as by constructional tolerances.

An alternate technique of scanning the antenna beam of a wide-angle microwave lens is the introduction of a linearly progressive, time-variable phase delay in the coaxial lens elements. This in turn causes a progressive phase shift between radiating probes of the straight front face of the lens and a corresponding shift in the position of the radiation pattern's peak. Thus, by putting phase shifters in the coaxial cables of the lens, a beam from a stationary input horn illuminator may be scanned over a limited region of space.

The original concept for the antenna studies described in this paper was that of a microwave lens that would combine a series of angularly spaced, stationary input horn illuminators with coaxial phase shifters in the lens elements. Although the beam generated through the lens by each horn points in a different direction, all the beams can be moved simultaneously by varying the phase shifters with progressively increasing phase. Thus, by spacing the horns at the proper angular

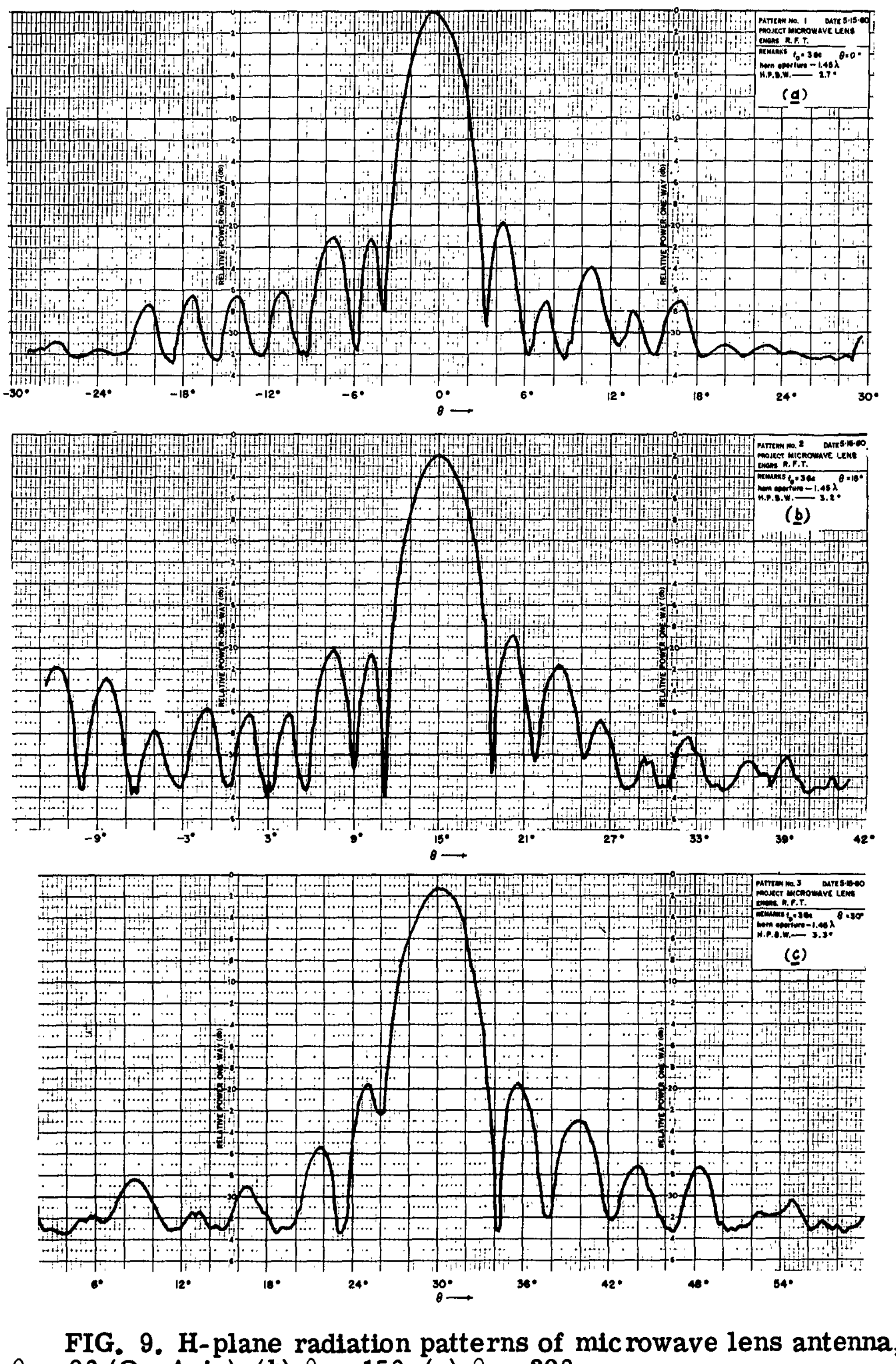

FIG. 9. H-plane radiation patterns of microwave lens antenna, (a) θ = 0° (On-Axis); (b) θ = 15°; (c) θ = 30°.

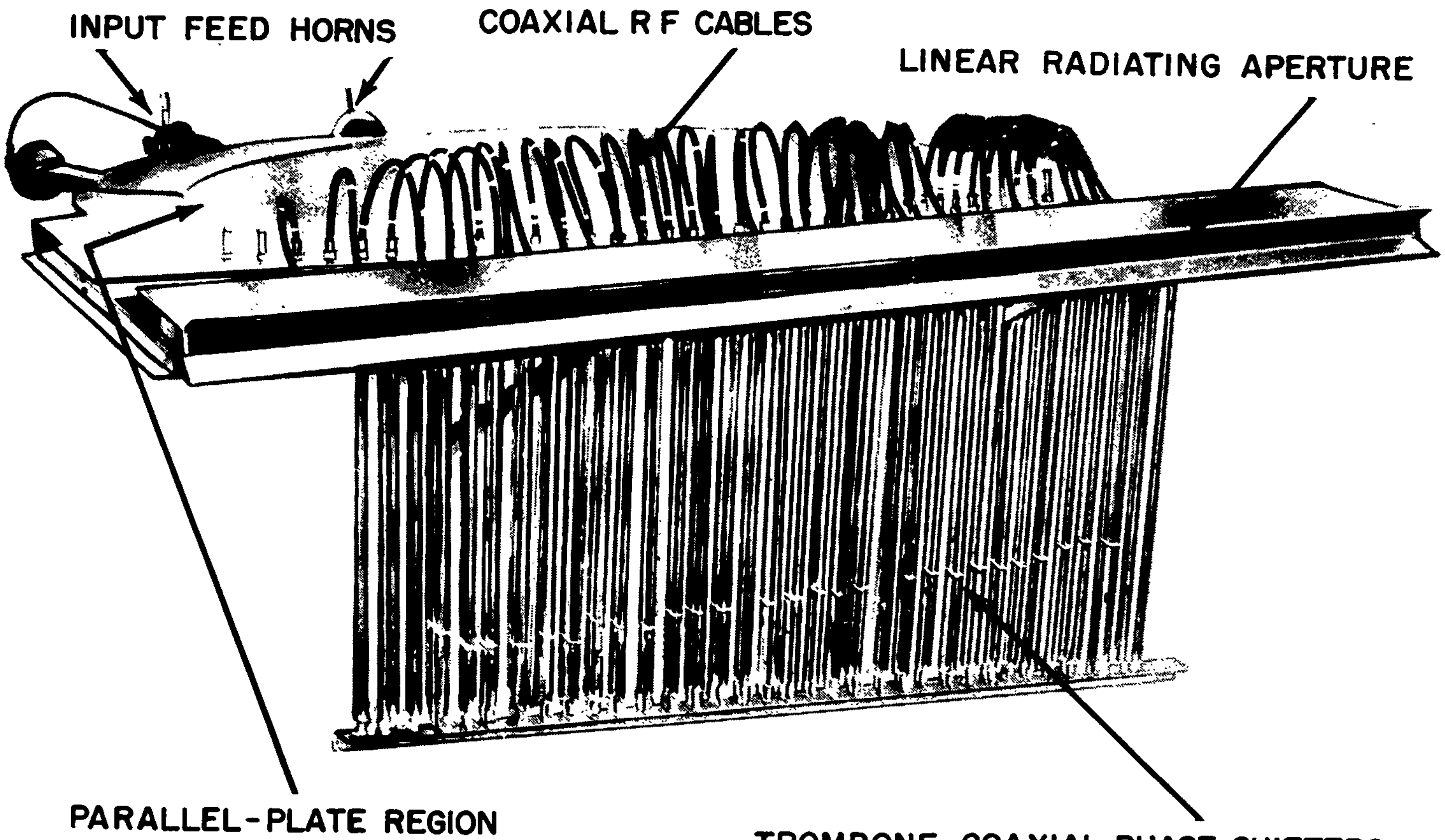

FIG. 10. Microwave lens antenna with phase shifters in lens elements.

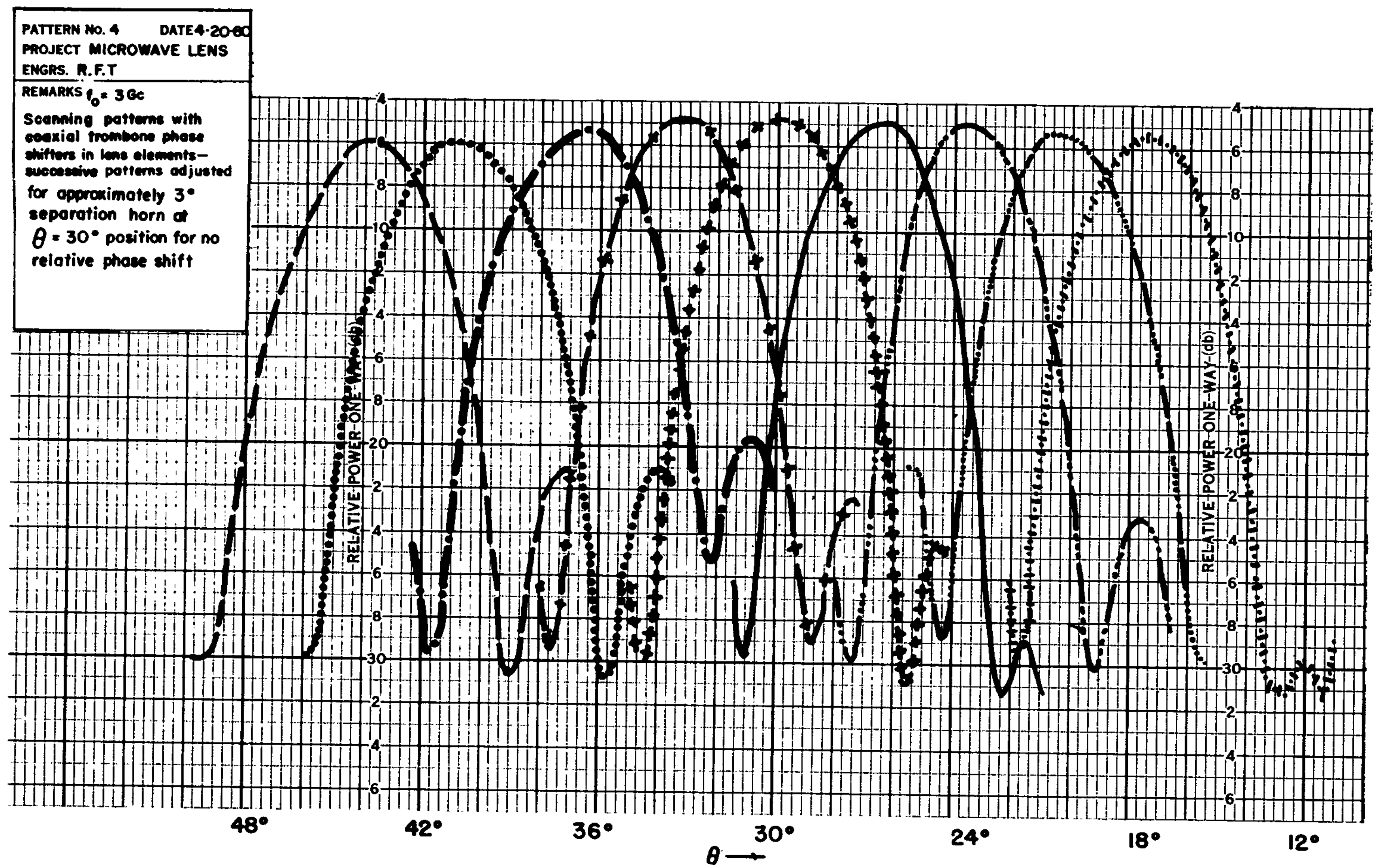

FIG. 11. Radiation patterns of microwave lens antenna with phase shifters in lens elements.

intervals, the entire field of view of the lens may be covered by several independent beams, each of which scans only a small sector. This antenna concept was designated as the Multiple-Beam Interval Scanner (MUBIS) system. Although its development has been delayed (replaced by a competitive antenna system[8] with a wide-angle microwave lens and coaxial organ-pipe scanners), a study was made of the action of the coaxial phase shifters in beam scanning. Figure 10 shows the wide-angle microwave lens model, modified by adding trombone-type coaxial phase shifters (line stretchers) to each of the coaxial cable lens elements. The phase shifters are adjusted to increase the cable lengths linearly along the face of the lens. A set of radiation patterns is shown in Fig. 11 for the case in which the input horn is located at 30°. The line stretchers are adjusted to scan the beam ± 12° about this central value in 3° steps. Only the region in the vicinity of the peak of each radiation pattern is shown. It can be seen that the gain of the antenna remains fairly constant and that the pattern shape does not deteriorate severely even for these large angles (42° maximum) from the normal. Similar patterns have been obtained with the input horn set at 0° and 15°.

4. CONCLUSIONS

The design principles presented in this report for two-dimensional lens structures may also be applied to three-dimensional lenses. The additional degree of freedom obtained by allowing nonuniform lens-element spacing ($y \neq \eta$), instead of the more conventional uniform spacing ($y = \eta$), permits the specification of two symmetrical off-axis and one on-axis focal points for a two-dimensional straight-front-face lens. In the Ruze ($y = \eta$) design the inner lens contour is uniquely determined by specifying the scan angle α. It is also known[1] that no solution is possible for the design of a three-dimensional, axially symmetrical lens if the Ruze constraints are applied. The inner contour of the Gent ($y \neq \eta$) two-dimensional lens design is however determined, not only by the required scan angle α, but also by the normalized on-axis focal length, g. Figure 4 shows that the shape of the inner lens contour depends strongly upon the parameter g.

A three-dimensional straight-front-face lens design may be derived as a figure of revolution of an appropriate two-dimensional lens contour. The parameter g that controls the inner lens contour should be selected to minimize the lens' astigmatism, as measured in the principal plane orthogonal to that of the selected focal points. A three-dimensional phase-error analysis would be required to assure that aberrations are within permissible tolerances over the entire lens surface.

It is also possible that the basic Gent lens equations may contain a solution for the design of a three-dimensional lens with two perfect and symmetrical off-axis focal points for which both lens surfaces can be nonplanar. Since these three-dimensional lens designs have not been extensively investigated under the present study, no numerical solutions are available.

The analysis presented in this paper has shown that microwave lenses that are capable of scanning wide-angle sectors with beamwidths on the order of a fraction of a degree are theoretically feasible and also practical to construct. These line sources may be used as primary feeds for parabolic cylindrical reflectors or planar arrays in large radar antenna systems.

Appendix A

LENS CONTOUR CALCULATIONS

The variables x, y, and w that specify the lens contour have been computed as a function of η for the straight-front-face lens design from Eqs. (4), (6), and (7) of the text and are presented below for the following range of parameters

$\alpha = 30°$,

$g = 1.137$; $\eta = 0 < 0.01 > 0.75$ (Table 1)

and

$g = 0.90 < 0.05 > 1.20$; $0 < 0.05 > 0.80$ (Table 2).

The normalized radius of curvature, $r = R/F$, of the focal arc is also given for each value of g.

The lens contour parameters for g = 1.00, 1.10, 1.137, and 1.200 are also shown graphically in Fig. 4.

Table 1.

Lens Contour Parameters for $g = 1.137$, $\alpha = 30°$, and $r = 0.597$

η	w	-x	y
0.00	0.00000	0.00000	0.00000
0.01	0.00000	0.00005	0.01000
0.02	0.00002	0.00019	0.02000

Table 1. (Contd)

η	w	-x	y
0.03	0.00004	0.00043	0.03000
0.04	0.00007	0.00077	0.04000
0.05	0.00011	0.00121	0.04999
0.06	0.00016	0.00174	0.05999
0.07	0.00021	0.00237	0.06999
0.08	0.00027	0.00309	0.07998
0.09	0.00034	0.00391	0.08997
0.10	0.00042	0.00483	0.09996
0.11	0.00051	0.00584	0.10994
0.12	0.00060	0.00695	0.11993
0.13	0.00070	0.00815	0.12991
0.14	0.00080	0.00945	0.13989
0.15	0.00091	0.01084	0.14986
0.16	0.00103	0.01233	0.15984
0.17	0.00115	0.01391	0.16980
0.18	0.00127	0.01559	0.17977
0.19	0.00140	0.01736	0.18973
0.20	0.00153	0.01922	0.19969
0.21	0.00166	0.02118	0.20965
0.22	0.00179	0.02323	0.21961
0.23	0.00192	0.02537	0.22956
0.24	0.00205	0.02761	0.23951
0.25	0.00217	0.02993	0.24946

Table 1. (Contd)

η	w	-x	y
0.26	0.00230	0.03234	0.25940
0.27	0.00241	0.03485	0.26935
0.28	0.00253	0.03744	0.27929
0.29	0.00263	0.04013	0.28924
0.30	0.00273	0.04290	0.29918
0.31	0.00281	0.04575	0.30913
0.32	0.00289	0.04870	0.31908
0.33	0.00294	0.05172	0.32903
0.34	0.00298	0.05484	0.33899
0.35	0.00301	0.05803	0.34895
0.36	0.00301	0.06130	0.35892
0.37	0.00298	0.06466	0.36890
0.38	0.00293	0.06809	0.37889
0.39	0.00284	0.07160	0.38890
0.40	0.00273	0.07519	0.39891
0.41	0.00257	0.07884	0.40895
0.42	0.00237	0.08257	0.41900
0.43	0.00213	0.08637	0.42909
0.44	0.00183	0.09023	0.43920
0.45	0.00147	0.09416	0.44934
0.46	0.00105	0.09814	0.45952
0.47	0.00056	0.10218	0.46974
0.48	-0.00001	0.10628	0.48000
0.49	-0.00066	0.11042	0.49032

Table 1. (Contd)

η	w	-x	y
0.50	-0.00142	0.11461	0.50071
0.51	-0.00227	0.11883	0.51116
0.52	-0.00325	0.12309	0.52169
0.53	-0.00435	0.12738	0.53231
0.54	-0.00560	0.13168	0.54302
0.55	-0.00701	0.13600	0.55385
0.56	-0.00859	0.14032	0.56481
0.57	-0.01038	0.14463	0.57592
0.58	-0.01238	0.14892	0.58718
0.59	-0.01464	0.15318	0.59864
0.60	-0.01717	0.15739	0.61030
0.61	-0.02001	0.16153	0.62221
0.62	-0.02321	0.16559	0.63439
0.63	-0.02681	0.16954	0.64689
0.64	-0.03086	0.17335	0.65975
0.65	-0.03543	0.17699	0.67303
0.66	-0.04060	0.18042	0.68679
0.67	-0.04645	0.18359	0.70112
0.68	-0.05910	0.18646	0.71611
0.69	-0.06068	0.18895	0.73187
0.70	-0.06935	0.19097	0.74855
0.71	-0.07932	0.19243	0.76632
0.72	-0.09085	0.19320	0.78541
0.73	-0.10427	0.19311	0.80611

Table 1. (Contd)

η	w	-x	y
0.74	-0.12000	0.19194	0.82880
0.75	-0.13861	0.18940	0.85395
0.80	-0.3172	0.1349	1.054

Table 2.

Lens Contour Parameters for g = 0.90<0.05>1.200 and $\alpha = 30°$

Table 2a. g = 0.900 (r = 3.70)

η	w	-x	y
0.00	-0.0004	0.0000	0.0000
0.05	0.0020	0.0034	0.0499
0.10	0.0079	0.0135	0.0992
0.15	0.0179	0.0302	0.1473
0.20	0.0318	0.0536	0.1936
0.25	0.0497	0.0836	0.2376
0.30	0.0717	0.1200	0.2785
0.35	0.0979	0.1626	0.3157
0.40	0.1282	0.2114	0.3487
0.45	0.1627	0.2662	0.3768
0.50	0.2015	0.3268	0.3993
0.55	0.2446	0.3931	0.4155
0.60	0.2920	0.4650	0.4248
0.65	0.3437	0.5429	0.4266
0.70	0.3991	0.6280	0.4206
0.75	0.4567	0.7254	0.4075
0.80	0.4988	0.8864	0.4009

Table 2b. g = 0.950 (r = 1.53)

η	w	-x	y
0.00	0.0000	0.0000	0.0000
0.05	0.0015	0.0028	0.0499
0.10	0.0060	0.0113	0.0994
0.15	0.0136	0.0254	0.1480
0.20	0.0243	0.0451	0.1951
0.25	0.0381	0.0704	0.2405
0.30	0.0551	0.1012	0.2835
0.35	0.0754	0.1375	0.3236
0.40	0.0991	0.1792	0.3604
0.45	0.1263	0.2262	0.3932
0.50	0.1572	0.2785	0.4214
0.55	0.1920	0.3360	0.4444
0.60	0.2308	0.3984	0.4615
0.65	0.2739	0.4659	0.4720
0.70	0.2312	0.5381	0.4752
0.75	0.3727	0.6154	0.4705
0.80	0.4272	0.6983	0.4583

Table 2c. g = 1.000 (r = 1.00)

η	w	-x	y
0.00	0.0000	0.0000	0.0000
0.05	0.0011	0.0023	0.0500
0.10	0.0043	0.0093	0.0996
0.15	0.0098	0.0210	0.1485
0.20	0.0175	0.0373	0.1965
0.25	0.0274	0.0583	0.2431
0.30	0.0397	0.0840	0.2881
0.35	0.0545	0.1143	0.3309
0.40	0.0718	0.1493	0.3713
0.45	0.0918	0.1889	0.4087
0.50	0.1146	0.2333	0.4427
0.55	0.1406	0.2822	0.4727
0.60	0.1699	0.3359	0.4981
0.65	0.2028	0.3942	0.5182
0.70	0.2399	0.4572	0.5321
0.75	0.2816	0.5248	0.5388
0.80	0.3285	0.5971	0.5372

Table 2d. g = 1.050 (r = 0.771)

η	w	-x	y
0.00	0.0008	0.0002	0.0000
0.05	0.0007	0.0019	0.0500
0.10	0.0028	0.0076	0.0997
0.15	0.0063	0.0170	0.1491
0.20	0.0112	0.0302	0.1978
0.25	0.0176	0.0473	0.2456
0.30	0.0255	0.0681	0.2924
0.35	0.0349	0.0927	0.3378
0.40	0.0458	0.1212	0.3817
0.45	0.0584	0.1534	0.4237
0.50	0.0726	0.1896	0.4637
0.55	0.0886	0.2296	0.5013
0.60	0.1064	0.2735	0.5362
0.65	0.1261	0.3213	0.5681
0.70	0.1476	0.3731	0.5967
0.75	0.1710	0.4287	0.6217
0.80	0.1957	0.4880	0.6434

Table 2<u>e</u>. g = 1.100 (r = 0.651)

η	w	-x	y
0.00	0.0009	0.0000	0.0000
0.05	0.0003	0.0015	0.0500
0.10	0.0014	0.0059	0.0999
0.15	0.0031	0.0134	0.1495
0.20	0.0055	0.0237	0.1989
0.25	0.0085	0.0370	0.2479
0.30	0.0121	0.0533	0.2964
0.35	0.0162	0.0724	0.3443
0.40	0.0208	0.0944	0.3917
0.45	0.0256	0.1191	0.4385
0.50	0.0304	0.1465	0.4848
0.55	0.0347	0.1765	0.5309
0.60	0.0381	0.2086	0.5772
0.65	0.0391	0.2424	0.6246
0.70	0.0354	0.2769	0.6753
0.75	0.0216	0.3097	0.7338
0.80	-0.0175	0.3346	0.8136

Table 2f. g = 1.150 (r = 0.582)

η	w	-x	y
0.00	0.0000	0.0000	0.0000
0.05	0.0000	0.0011	0.0500
0.10	0.0001	0.0045	0.1000
0.15	0.0002	0.0100	0.1500
0.20	0.0002	0.0177	0.2000
0.25	0.0000	0.0275	0.2500
0.30	-0.0005	0.0394	0.3001
0.35	-0.0015	0.0531	0.3505
0.40	-0.0035	0.0686	0.4014
0.45	-0.0070	0.0855	0.4531
0.50	-0.0127	0.1033	0.5064
0.55	-0.0222	0.1214	0.5622
0.60	-0.0378	0.1385	0.6227
0.65	-0.0644	0.1520	0.6918
0.70	-0.1126	0.1562	0.7789
0.75	-0.2127	0.1352	0.9095
0.80	-0.4990	0.0181	1.199

Table 2g. $g = 1.200$ ($r = 0.541$)

η	w	-x	y
.00	+0.0006	0.0003	0.0000
.05	-0.0003	0.0008	0.0500
.10	-0.0011	0.0031	0.1001
.15	-0.0025	0.0069	0.1504
.20	-0.0048	0.0121	0.2010
.25	-0.0079	0.0186	0.2520
.30	-0.0124	0.0263	0.3037
.35	-0.0186	0.0347	0.3565
.40	-0.0273	0.0435	0.4109
.45	-0.0396	0.0521	0.4678
.50	-0.0572	0.0593	0.5286
.55	-0.0836	0.0632	0.5960
.60	-0.1247	0.0601	0.6748
.65	-0.1944	0.0417	0.7764
.70	-0.3307	-0.0146	0.9315
.75	-0.6998	-0.2085	1.275
.80	-25.45	-15.00	21.16

Appendix B

PHASE ERROR CALCULATIONS

The normalized path length errors, $\Delta \ell$, that specify the aberrations of the lens, have been computed from Eq. (9) and the lens contour parameters and are given below for the following ranges of variables

$\alpha = 30°$

$\eta = 0.00 < 0.05 > 0.80$,

$g = 0.90 < 0.05 > 1.20$ (Table 3.),

and

$g = 1.137$ (Table 4.),

$\theta = \pm 5°, \pm 10°, \pm 15°, \pm 20°, \pm 25°, \pm 35°, \pm 40°$.

The values of $\Delta \ell$ for $\theta = \pm\alpha\,(\pm 30°)$ and $\theta = 0°$ are zero.

Curves of $\Delta \ell$ versus η are shown in Fig. 5 for $g = 1.00$, 1.10, and 1.137.

Note that only positive values of θ are plotted in Fig. 5 with both positive and negative values of η. In Tables 3 and 4, the equivalent representation of only positive values of η are shown with both positive and negative values of θ.

Table 3.

Normalized Path Length Errors, $\Delta \ell$, for Microwave Lens $\alpha = 30°$

Table 3a. g = 0.900

η	$\theta = 5°$	$\theta = 10°$	$\theta = 15°$	$\theta = 20°$	$\theta = 25°$	$\theta = 35°$	$\theta = 40°$
0.00	-0.000000	-0.000000	-0.000000	-0.000000	-0.000000	-0.000000	0.000000
0.05	-0.000003	-0.000008	-0.000014	-0.000017	-0.000014	-0.000030	0.000081
0.10	-0.000015	-0.000039	-0.000065	-0.000080	-0.000065	-0.000135	0.000360
0.15	-0.000042	-0.000106	-0.000172	-0.000204	-0.000164	-0.000337	0.000896
0.20	-0.000092	-0.000223	-0.000352	-0.000415	-0.000329	-0.000668	0.001767
0.25	-0.000173	-0.000408	-0.000632	-0.000737	-0.000580	-0.001168	0.003076
0.30	-0.000296	-0.000682	-0.001044	-0.001207	-0.000945	-0.001892	0.004962
0.35	-0.000473	-0.001074	-0.001630	-0.001876	-0.001464	-0.002914	0.007615
0.40	-0.000721	-0.001622	-0.002450	-0.002811	-0.002191	-0.004345	0.011310
0.45	-0.001064	-0.002380	-0.003587	-0.004116	-0.003210	-0.006351	0.016460
0.50	-0.001534	-0.003425	-0.005167	-0.005946	-0.004650	-0.009192	0.023700
0.55	-0.002173	-0.004872	-0.007389	-0.008557	-0.006728	-0.013300	0.034040
0.60	-0.003066	-0.006907	-0.010580	-0.012390	-0.009839	-0.019450	0.049160
0.65	-0.004307	-0.009836	-0.015340	-0.018330	-0.014790	-0.029120	0.071970
0.70	-0.006077	-0.014210	-0.022870	-0.028300	-0.023510	-0.045350	0.107300
0.75	-0.008639	-0.021020	-0.035870	-0.047920	-0.042440	-0.074340	0.162000
0.80	-0.011520	-0.029640	-0.055880	-0.091820	-0.118800	-0.116600	0.228300

Table 3a. (Contd)

η	θ = -5°	θ = -10°	θ = -15°	θ = -20°	θ = -25°	θ = -35°	θ = -40°
0.00	0.000000	-0.000000	-0.000000	-0.000000	-0.000000	0.000000	0.000000
0.05	-0.000001	-0.000004	-0.000009	-0.000013	-0.000011	0.000024	0.000067
0.10	0.000001	-0.000011	-0.000029	-0.000042	-0.000038	0.000087	0.000241
0.15	0.000010	-0.000010	-0.000046	-0.000076	-0.000072	0.000175	0.000489
0.20	0.000033	0.000010	-0.000048	-0.000103	-0.000106	0.000275	0.000780
0.25	0.000077	0.000059	-0.000022	-0.000110	-0.000131	0.000375	0.001085
0.30	0.000149	0.000150	0.000045	-0.000086	-0.000141	0.000464	0.001379
0.35	0.000259	0.000296	0.000169	-0.000018	-0.000127	0.000532	0.001636
0.40	0.000417	0.000516	0.000369	0.000111	-0.000080	0.000565	0.001829
0.45	0.000639	0.000830	0.000666	0.000318	0.000012	0.000551	0.001930
0.50	0.000943	0.001265	0.001087	0.000625	0.000159	0.000475	0.001908
0.55	0.001355	0.001858	0.001667	0.001058	0.000376	0.000322	0.001730
0.60	0.001910	0.002654	0.002450	0.001651	0.000681	0.000073	0.001357
0.65	0.002657	0.003714	0.003490	0.002442	0.001095	-0.000293	0.000751
0.70	0.003661	0.005118	0.004854	0.003476	0.001638	-0.000793	-0.000122
0.75	0.004992	0.006930	0.006584	0.004773	0.002315	-0.001420	-0.001236
0.80	0.006284	0.008593	0.008086	0.005834	0.002830	-0.001791	-0.001736

Table 3b. g = 0.950

η	θ = 5°	θ = 10°	θ = 15°	θ = 20°	θ = 25°	θ = 35°	θ = 40°
0.00	-0.000000	-0.000000	-0.000000	-0.000000	-0.000000	0.000000	0.000000
0.05	-0.000001	-0.000005	-0.000009	-0.000011	-0.000009	0.000020	0.000054
0.10	-0.000007	-0.000022	-0.000038	-0.000048	-0.000040	0.000086	0.000232
0.15	-0.000020	-0.000056	-0.000095	-0.000118	-0.000097	0.000208	0.000563
0.20	-0.000042	-0.000112	-0.000188	-0.000230	-0.000188	0.000401	0.001083
0.25	-0.000075	-0.000197	-0.000324	-0.000394	-0.000321	0.000682	0.001843
0.30	-0.000125	-0.000318	-0.000517	-0.000624	-0.000507	0.001076	0.002906
0.35	-0.000195	-0.000484	-0.000780	-0.000939	-0.000760	0.001616	0.004364
0.40	-0.000290	-0.000709	-0.001135	-0.001361	-0.001102	0.002347	0.006345
0.45	-0.000418	-0.001009	-0.001607	-0.001926	-0.001561	0.003339	0.009040
0.50	-0.000588	-0.001408	-0.002235	-0.002682	-0.002180	0.004696	0.012740
0.55	-0.000813	-0.001936	-0.003075	-0.003702	-0.003025	0.006585	0.017910
0.60	-0.001111	-0.002641	-0.004209	-0.005101	-0.004204	0.009291	0.025340
0.65	-0.001510	-0.003595	-0.005771	-0.007071	-0.005905	0.013340	0.036460
0.70	-0.002043	-0.004910	-0.007984	-0.009959	-0.008490	0.019780	0.053960
0.75	-0.002786	-0.006771	-0.011250	-0.014450	-0.012750	0.031000	0.083200
0.80	-0.003847	-0.009471	-0.016320	-0.022070	-0.020770	0.052880	0.133600

Table 3b. (Contd)

η	θ = -5°	θ = -10°	θ = -15°	θ = -20°	θ = -25°	θ = -35°	θ = -40°
0.00	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.05	-0.000001	-0.000003	-0.000007	-0.000009	-0.000008	0.000017	0.000047
0.10	-0.000002	-0.000011	-0.000024	-0.000033	-0.000029	0.000065	0.000177
0.15	-0.000001	-0.000020	-0.000048	-0.000067	-0.000059	0.000136	0.000373
0.20	0.000004	-0.000026	-0.000073	-0.000108	-0.000097	0.000226	0.000624
0.25	0.000014	-0.000026	-0.000094	-0.000150	-0.000138	0.000330	0.000917
0.30	0.000034	-0.000015	-0.000108	-0.000189	-0.000181	0.000444	0.001242
0.35	0.000065	0.000012	-0.000110	-0.000221	-0.000222	0.000564	0.001589
0.40	0.000112	0.000059	-0.000093	-0.000241	-0.000259	0.000686	0.001950
0.45	0.000178	0.000133	-0.000051	-0.000245	-0.000288	0.000806	0.002315
0.50	0.000269	0.000243	0.000024	-0.000225	-0.000306	0.000919	0.002674
0.55	0.000393	0.000397	0.000142	-0.000173	-0.000308	0.001020	0.003017
0.60	0.000559	0.000609	0.000316	-0.000080	-0.000289	0.001105	0.003334
0.65	0.000780	0.000897	0.000560	0.000065	-0.000244	0.001166	0.003611
0.70	0.001073	0.001281	0.000895	0.000276	-0.000165	0.001197	0.003837
0.75	0.001462	0.001791	0.001344	0.000570	-0.000044	0.001192	0.003998
0.80	0.001973	0.002451	0.001926	0.000956	0.000121	0.001150	0.004098

Table 3c. g = 1.000

η	θ = 5°	θ = 10°	θ = 15°	θ = 20°	θ = 25°	θ = 35°	θ = 40°
0.00	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.05	-0.000001	-0.000002	-0.000004	-0.000006	-0.000005	0.000011	0.000030
0.10	-0.000003	-0.000009	-0.000018	-0.000023	-0.000020	0.000045	0.000126
0.15	-0.000006	-0.000021	-0.000040	-0.000053	-0.000046	0.000106	0.000294
0.20	-0.000010	-0.000038	-0.000073	-0.000097	-0.000084	0.000195	0.000548
0.25	-0.000016	-0.000061	-0.000117	-0.000157	-0.000136	0.000319	0.000900
0.30	-0.000024	-0.000089	-0.000174	-0.000233	-0.000204	0.000484	0.001370
0.35	-0.000033	-0.000125	-0.000244	-0.000330	-0.000290	0.000697	0.001986
0.40	-0.000044	-0.000168	-0.000330	-0.000449	-0.000398	0.000970	0.002783
0.45	-0.000058	-0.000220	-0.000435	-0.000597	-0.000532	0.001319	0.003815
0.50	-0.000074	-0.000283	-0.000563	-0.000778	-0.000700	0.001768	0.005161
0.55	-0.000092	-0.000357	-0.000718	-0.001002	-0.000911	0.002352	0.006946
0.60	-0.000115	-0.000448	-0.000909	-0.001281	-0.001179	0.003130	0.009374
0.65	-0.000141	-0.000557	-0.001144	-0.001635	-0.001528	0.004199	0.012810
0.70	-0.000174	-0.000693	-0.001442	-0.002095	-0.001996	0.005746	0.017960
0.75	-0.000213	-0.000864	-0.001829	-0.002713	-0.002653	0.008156	0.026370
0.80	-0.000264	-0.001085	-0.002352	-0.003590	-0.003643	0.012400	0.042000

Table 3c (Contd)

η	θ = -5°	θ =-10°	θ = -15°	θ = -20°	θ = -25°	θ = -35°	θ = -40°
0.00	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.05	-0.000001	-0.000002	-0.000004	-0.000005	-0.000005	0.000010	0.000028
0.10	-0.000002	-0.000009	-0.000017	-0.000022	-0.000018	0.000040	0.000110
0.15	-0.000006	-0.000020	-0.000037	-0.000048	-0.000040	0.000089	0.000242
0.20	-0.000010	-0.000035	-0.000066	-0.000085	-0.000071	0.000155	0.000421
0.25	-0.000016	-0.000056	-0.000103	-0.000131	-0.000109	0.000237	0.000645
0.30	-0.000023	-0.000080	-0.000148	-0.000188	-0.000156	0.000337	0.000913
0.35	-0.000031	-0.000110	-0.000201	-0.000256	-0.000211	0.000453	0.001223
0.40	-0.000041	-0.000145	-0.000264	-0.000334	-0.000275	0.000586	0.001578
0.45	-0.000053	-0.000185	-0.000336	-0.000424	-0.000348	0.000736	0.001976
0.50	-0.000067	-0.000232	-0.000419	-0.000526	-0.000430	0.000904	0.002420
0.55	-0.000083	-0.000286	-0.000514	-0.000642	-0.000523	0.001091	0.002913
0.60	-0.000101	-0.000348	-0.000622	-0.000773	-0.000627	0.001299	0.003458
0.65	-0.000123	-0.000419	-0.000745	-0.000921	-0.000744	0.001530	0.004060
0.70	-0.000148	-0.000502	-0.000886	-0.001090	-0.000876	0.001787	0.004727
0.75	-0.000178	-0.000598	-0.001050	-0.001283	-0.001026	0.002074	0.005468
0.80	-0.000214	-0.000713	-0.001241	-0.001506	-0.001197	0.002397	0.006296

Table 3d. g = 1.050

η	θ = 5°	θ = 10°	θ = 15°	θ = 20°	θ = 25°	θ = 35°	θ = 40°
0.00	0.000000	-0.000000	-0.000000	-0.000000	-0.000000	0.000000	0.000000
0.05	0.000000	-0.000001	-0.000001	-0.000002	-0.000002	0.000004	0.000012
0.10	0.000000	-0.000001	-0.000004	-0.000006	-0.000006	0.000016	0.000046
0.15	0.000002	-0.000001	-0.000006	-0.000011	-0.000012	0.000033	0.000099
0.20	0.000005	0.000003	-0.000006	-0.000015	-0.000017	0.000054	0.000168
0.25	0.000012	0.000012	-0.000000	-0.000015	-0.000022	0.000078	0.000250
0.30	0.000024	0.000028	0.000014	-0.000009	-0.000023	0.000101	0.000338
0.35	0.000040	0.000053	0.000038	0.000007	-0.000019	0.000121	0.000427
0.40	0.000064	0.000090	0.000077	0.000036	-0.000007	0.000134	0.000509
0.45	0.000096	0.000143	0.000135	0.000082	0.000017	0.000134	0.000574
0.50	0.000139	0.000215	0.000217	0.000152	0.000055	0.000115	0.000606
0.55	0.000194	0.000310	0.000329	0.000252	0.000114	0.000068	0.000585
0.60	0.000264	0.000435	0.000480	0.000391	0.000200	-0.000021	0.000477
0.65	0.000353	0.000596	0.000679	0.000580	0.000323	-0.000171	0.000235
0.70	0.000464	0.000801	0.000939	0.000836	0.000494	-0.000411	-0.000222
0.75	0.000601	0.001058	0.001274	0.001174	0.000730	-0.000787	-0.001031
0.80	0.000767	0.001376	0.001696	0.001613	0.001049	-0.001362	-0.002428

Table 3d. (Contd)

η	θ = -5°	θ = -10°	θ = -15°	θ = -20°	θ = -25°	θ = -35°	θ = -40°
0.00	-0.000000	-0.000000	-0.000000	-0.000000	-0.000000	0.000000	0.000000
0.05	-0.000000	-0.000001	-0.000002	-0.000002	-0.000002	0.000005	0.000013
0.10	-0.000002	-0.000005	-0.000009	-0.000011	-0.000009	0.000019	0.000053
0.15	-0.000006	-0.000014	-0.000023	-0.000027	-0.000021	0.000045	0.000123
0.20	-0.000013	-0.000030	-0.000045	-0.000052	-0.000041	0.000083	0.000225
0.25	-0.000024	-0.000052	-0.000078	-0.000088	-0.000068	0.000136	0.000362
0.30	-0.000039	-0.000085	-0.000123	-0.000137	-0.000104	0.000203	0.000537
0.35	-0.000061	-0.000128	-0.000183	-0.000200	-0.000150	0.000287	0.000753
0.40	-0.000090	-0.000185	-0.000259	-0.000279	-0.000207	0.000389	0.001012
0.45	-0.000127	-0.000257	-0.000355	-0.000377	-0.000277	0.000511	0.001319
0.50	-0.000175	-0.000347	-0.000472	-0.000497	-0.000361	0.000655	0.001678
0.55	-0.000234	-0.000458	-0.000615	-0.000640	-0.000460	0.000823	0.002094
0.60	-0.000307	-0.000593	-0.000787	-0.000810	-0.000578	0.001017	0.002571
0.65	-0.000396	-0.000756	-0.000992	-0.001011	-0.000715	0.001240	0.003116
0.70	-0.000504	-0.000951	-0.001234	-0.001247	-0.000874	0.001495	0.003734
0.75	-0.000634	-0.001181	-0.001518	-0.001520	-0.001057	0.001784	0.004429
0.80	-0.000786	-0.001448	-0.001844	-0.001831	-0.001264	0.002106	0.005200

Table 3e. g = 1.100

η	θ = 5°	θ = 10°	θ = 15°	θ = 20°	θ = 25°	θ = 35°	θ = 40°
0.00	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.05	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000001
0.10	0.000001	0.000002	0.000002	0.000002	0.000001	-0.000001	-0.000000
0.15	0.000003	0.000006	0.000007	0.000007	0.000004	-0.000005	-0.000009
0.20	0.000008	0.000015	0.000018	0.000017	0.000011	-0.000016	-0.000033
0.25	0.000016	0.000029	0.000037	0.000035	0.000024	-0.000035	-0.000078
0.30	0.000028	0.000051	0.000065	0.000063	0.000043	-0.000067	-0.000154
0.35	0.000045	0.000082	0.000104	0.000103	0.000070	-0.000113	-0.000268
0.40	0.000066	0.000122	0.000156	0.000155	0.000107	-0.000177	-0.000430
0.45	0.000092	0.000172	0.000221	0.000222	0.000154	-0.000262	-0.000645
0.50	0.000123	0.000230	0.000297	0.000300	0.000211	-0.000365	-0.000913
0.55	0.000156	0.000292	0.000380	0.000387	0.000274	-0.000482	-0.001223
0.60	0.000187	0.000352	0.000460	0.000469	0.000334	-0.000596	-0.001529
0.65	0.000208	0.000392	0.000511	0.000522	0.000372	-0.000666	-0.001711
0.70	0.000201	0.000374	0.000483	0.000488	0.000342	-0.000593	-0.001482
0.75	0.000123	0.000216	0.000260	0.000240	0.000148	-0.000152	-0.000163
0.80	-0.000129	-0.000292	-0.000452	-0.000546	-0.000462	0.001201	0.003835

Table 3e. (Contd)

η	θ = -5°	θ = -10°	θ = -15°	θ = -20°	θ = -25°	θ = -35°	θ = -40°
0.00	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.05	0.000000	0.000000	0.000000	-0.000001	0.000000	0.000001	0.000002
0.10	-0.000001	-0.000002	-0.000003	-0.000003	-0.000002	0.000005	0.000013
0.15	-0.000004	-0.000007	-0.000009	-0.000010	-0.000007	0.000013	0.000035
0.20	-0.000009	-0.000016	-0.000021	-0.000022	-0.000015	0.000028	0.000072
0.25	-0.000017	-0.000031	-0.000040	-0.000040	-0.000028	0.000050	0.000128
0.30	-0.000028	-0.000052	-0.000067	-0.000067	-0.000046	0.000080	0.000205
0.35	-0.000044	-0.000081	-0.000103	-0.000102	-0.000070	-0.000120	0.000303
0.40	-0.000065	-0.000118	-0.000148	-0.000146	-0.000101	0.000169	0.000424
0.45	-0.000090	-0.000162	-0.000203	-0.000199	-0.000136	0.000227	0.000566
0.50	-0.000118	-0.000213	-0.000266	-0.000260	-0.000177	0.000293	0.000727
0.55	-0.000149	-0.000267	-0.000333	-0.000324	-0.000221	0.000363	0.000897
0.60	-0.000178	-0.000319	-0.000397	-0.000386	-0.000263	0.000430	0.001062
0.65	-0.000199	-0.000356	-0.000443	-0.000432	-0.000294	0.000483	0.001192
0.70	-0.000194	-0.000351	-0.000440	-0.000432	-0.000296	0.000493	0.001227
0.75	-0.000131	-0.000246	-0.000319	-0.000324	-0.000229	0.000404	0.001033
0.80	0.000081	0.000110	0.000096	0.000055	0.000011	0.000066	0.000267

Table 3f. g = 1.150

η	θ = 5°	θ = 10°	θ = 15°	θ = 20°	θ = 25°	θ = 35°	θ = 40°
0.00	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.05	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	-0.000001
0.10	0.000000	0.000000	0.000000	0.000000	0.000000	-0.000001	-0.000002
0.15	0.000000	0.000001	0.000001	0.000001	0.000001	-0.000002	-0.000004
0.20	0.000000	0.000001	0.000001	0.000001	0.000001	-0.000002	-0.000006
0.25	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.30	-0.000002	-0.000004	-0.000005	-0.000006	-0.000004	0.000009	0.000026
0.35	-0.000007	-0.000013	-0.000018	-0.000020	-0.000015	0.000032	0.000092
0.40	-0.000017	-0.000034	-0.000048	-0.000052	-0.000040	0.000084	0.000238
0.45	-0.000038	-0.000075	-0.000105	-0.000114	-0.000088	0.000181	0.000531
0.50	-0.000076	-0.000152	-0.000211	-0.000230	-0.000177	0.000378	0.001088
0.55	-0.000144	-0.000288	-0.000401	-0.000439	-0.000337	0.000728	0.002111
0.60	-0.000264	-0.000529	-0.000736	-0.000807	-0.000621	0.001352	0.003946
0.65	-0.000477	-0.000954	-0.001330	-0.001458	-0.001125	0.002458	0.007195
0.70	-0.000864	-0.001727	-0.002405	-0.002636	-0.002032	0.004422	0.012890
0.75	-0.001616	-0.003219	-0.004467	-0.004872	-0.003733	0.007951	0.022710
0.80	-0.003314	-0.006540	-0.008964	-0.009628	-0.007233	0.014510	0.039610

Table 3f. (Contd)

η	θ = -5°	θ = -10°	θ = -15°	θ = -20°	θ = -25°	θ = -35°	θ = -40°
0.00	0.000000	-0.000000	-0.000000	-0.000000	0.000000	0.000000	0.000000
0.05	0.000000	-0.000000	-0.000000	-0.000000	0.000000	0.000000	0.000000
0.10	0.000000	-0.000000	-0.000000	-0.000000	0.000000	0.000000	-0.000001
0.15	0.000000	-0.000000	-0.000000	-0.000000	0.000000	0.000000	0.000000
0.20	0.000000	-0.000001	-0.000001	-0.000001	0.000000	0.000000	0.000001
0.25	0.000000	-0.000000	-0.000000	-0.000000	0.000000	0.000000	0.000001
0.30	0.000001	0.000002	0.000003	0.000002	0.000001	-0.000001	-0.000003
0.35	0.000005	0.000009	0.000010	0.000009	0.000006	-0.000007	-0.000015
0.40	0.000014	0.000024	0.000028	0.000025	0.000016	-0.000021	-0.000045
0.45	0.000032	0.000053	0.000062	0.000056	0.000036	-0.000049	-0.000108
0.50	0.000064	0.000108	0.000126	0.000115	0.000073	-0.000102	-0.000230
0.55	0.000122	0.000206	0.000242	0.000221	0.000141	-0.000199	-0.000451
0.60	0.000224	0.000379	0.000445	0.000408	0.000260	-0.000371	-0.000845
0.65	0.000405	0.000685	0.000805	0.000739	0.000473	-0.000677	-0.001548
0.70	0.000734	0.001244	0.001463	0.001345	0.000861	-0.001239	-0.002840
0.75	0.001379	0.002342	0.002758	0.002540	0.001630	-0.002352	-0.005404
0.80	0.002871	0.004901	0.005799	0.005360	0.003450	-0.005003	-0.011510

Table 3g. g = 1.200

η	θ = 5°	θ = 10°	θ = 15°	θ = 20°	θ = 25°	θ = 35°	θ = 40°
0.00	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.05	0.000000	-0.000001	-0.000002	-0.000002	-0.000002	0.000004	0.000014
0.10	-0.000002	-0.000005	-0.000008	-0.000010	-0.000008	0.000020	0.000066
0.15	-0.000007	-0.000015	-0.000023	-0.000028	-0.000023	0.000056	0.000175
0.20	-0.000016	-0.000035	-0.000052	-0.000061	-0.000049	0.000120	0.000372
0.25	-0.000033	-0.000069	-0.000102	-0.000117	-0.000094	0.000225	0.000698
0.30	-0.000060	-0.000125	-0.000182	-0.000207	-0.000166	0.000392	0.001217
0.35	-0.000103	-0.000213	-0.000307	-0.000347	-0.000276	0.000653	0.002025
0.40	-0.000169	-0.000348	-0.000499	-0.000562	-0.000446	0.001051	0.003268
0.45	-0.000272	-0.000555	-0.000792	-0.000889	-0.000705	0.001659	0.005165
0.50	-0.000428	-0.000872	-0.001239	-0.001388	-0.001099	0.002585	0.008053
0.55	-0.000671	-0.001362	-0.001929	-0.002157	-0.001704	0.003999	0.012440
0.60	-0.001054	-0.002134	-0.003016	-0.003363	-0.002650	0.006172	0.019060
0.65	-0.001685	-0.003399	-0.004785	-0.005314	-0.004166	0.009546	0.029020
0.70	-0.002798	-0.005612	-0.007849	-0.008647	-0.006710	0.014860	0.043850
0.75	-0.005063	-0.010040	-0.013840	-0.014990	-0.011370	0.023540	0.065970
0.80	-0.013460	-0.025470	-0.033260	-0.033820	-0.023890	0.041660	0.105600

Table 3g. (Contd)

η	θ = -5°	θ = -10°	θ = -15°	θ = -20°	θ = -25°	θ = -35°	θ = -40°
0.00	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.05	0.000000	0.000000	-0.000001	-0.000001	-0.000001	0.000003	0.000010
0.10	0.000001	0.000001	-0.000001	-0.000002	-0.000003	0.000009	0.000031
0.15	0.000004	0.000005	0.000003	0.000000	-0.000003	0.000015	0.000057
0.20	0.000011	0.000015	0.000013	0.000007	0.000001	0.000017	0.000075
0.25	0.000023	0.000035	0.000035	0.000025	0.000010	0.000011	0.000077
0.30	0.000045	0.000070	0.000074	0.000059	0.000030	-0.000009	0.000047
0.35	0.000080	0.000128	0.000139	0.000116	0.000065	-0.000051	-0.000034
0.40	0.000135	0.000219	0.000244	0.000210	0.000123	-0.000126	-0.000192
0.45	0.000219	0.000360	0.000408	0.000359	0.000216	-0.000253	-0.000469
0.50	0.000350	0.000579	0.000664	0.000591	0.000364	-0.000460	-0.000932
0.55	0.000553	0.000922	0.001064	0.000958	0.000598	-0.000793	-0.001690
0.60	0.000876	0.001467	0.001704	0.001546	0.000975	-0.001336	-0.002937
0.65	0.001410	0.002373	0.002771	0.002528	0.001606	-0.002252	-0.005055
0.70	0.002364	0.003996	0.004689	0.004300	0.002748	-0.003920	-0.008922
0.75	0.004354	0.007413	0.008752	0.008073	0.005188	-0.007495	-0.017210
0.80	0.012440	0.021760	0.026230	0.024570	0.015940	-0.023130	-0.052810

Table 4.

Normalized Path Length Errors $\Delta \ell$, for Microwave Lens - $\alpha = 30°$ and $g = 1.137$.

η	$\theta = -5°$	$\theta = -10°$	$\theta = -15°$	$\theta = -20°$	$\theta = -25°$	$\theta = -35°$	$\theta = -40°$
0.00	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.05	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	-0.000001
0.10	0.000000	-0.000001	-0.000001	0.000000	0.000000	0.000000	0.000000
0.15	-0.000001	-0.000002	-0.000002	-0.000002	-0.000001	0.000001	0.000002
0.20	-0.000003	-0.000005	-0.000005	-0.000005	-0.000003	0.000004	0.000009
0.25	-0.000005	-0.000009	-0.000010	-0.000009	-0.000006	0.000008	0.000020
0.30	-0.000008	-0.000013	-0.000016	-0.000015	-0.000010	0.000014	0.000034
0.35	-0.000010	-0.000018	-0.000021	-0.000020	-0.000013	0.000020	0.000048
0.40	-0.000011	-0.000019	-0.000023	-0.000022	-0.000014	0.000023	0.000057
0.45	-0.000007	-0.000012	-0.000016	-0.000015	-0.000010	0.000019	0.000049
0.50	0.000006	0.000009	0.000009	0.000007	0.000003	0.000000	0.000008
0.55	0.000036	0.000059	0.000067	0.000060	0.000037	-0.000047	-0.000098
0.60	0.000095	0.000160	0.000186	0.000168	0.000106	-0.000145	-0.000322
0.65	0.000210	0.000354	0.000414	0.000378	0.000240	-0.000337	-0.000761
0.70	0.000431	0.000729	0.000855	0.000783	0.000500	-0.000710	-0.001616
0.75	0.000875	0.001483	0.001743	0.001601	0.001024	-0.001467	-0.003355
0.80	0.001883	0.003205	0.003781	0.003485	0.002237	-0.003226	-0.007403

η	$\theta = 5°$	$\theta = 10°$	$\theta = 15°$	$\theta = 20°$	$\theta = 25°$	$\theta = 35°$	$\theta = 40°$
0.00	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.05	0.000000	0.000000	0.000000	0.000000	0.000000	-0.000001	-0.000001
0.10	0.000000	0.000001	0.000002	0.000002	0.000001	-0.000003	-0.000007
0.15	0.000002	0.000003	0.000004	0.000005	0.000004	-0.000007	-0.000019
0.20	0.000003	0.000007	0.000009	0.000010	0.000008	-0.000015	-0.000040
0.25	0.000006	0.000012	0.000016	0.000017	0.000013	-0.000025	-0.000068
0.30	0.000009	0.000018	0.000024	0.000025	0.000019	-0.000037	-0.000099
0.35	0.000011	0.000022	0.000030	0.000032	0.000024	-0.000046	-0.000123
0.40	0.000012	0.000023	0.000030	0.000032	0.000023	-0.000045	-0.000119
0.45	0.000007	0.000013	0.000016	0.000016	0.000011	-0.000019	-0.000045
0.50	-0.000008	-0.000018	-0.000027	-0.000031	-0.000025	0.000058	0.000175
0.55	-0.000043	-0.000088	-0.000123	-0.000136	-0.000106	0.000233	0.000680
0.60	-0.000114	-0.000228	-0.000319	-0.000351	-0.000271	0.000591	0.001725
0.65	-0.000249	-0.000499	-0.000697	-0.000765	-0.000591	0.001290	0.003772
0.70	-0.000510	-0.001020	-0.001422	-0.001560	-0.001204	0.002624	0.007659
0.75	-0.001029	-0.002055	-0.002858	-0.003125	-0.002402	0.005164	0.014880
0.80	-0.002191	-0.004344	-0.005988	-0.006476	-0.004907	0.010100	0.028070

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